BRAILLE AUTHORITY OF THE UNITED KINGDOM

BRAILLE MATHEMATICS NOTATION

Royal National Institute of Blind People
Bakewell Road, Orton Southgate
Peterborough, Cambridgeshire
PE2 6XU

2005

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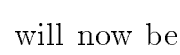
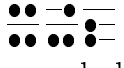
INTRODUCTION TO 1987 EDITION

This edition of the *Braille Mathematics Notation* has been prepared by the Mathematics Committee of the Braille Authority of the United Kingdom. It arose from the need to extend and amend the 1980 edition (in print, the 1970 edition with 1979 Addendum), and this also seemed to be a good occasion to rewrite the code book in a more convenient form. It should be emphasized that no changes have been made in such basic notations as those for addition, subtraction, multiplication, division, brackets, equality, unions, intersections, inequalities and so on.

Two of the most important changes made in this edition are concerned with unit abbreviations and with letter fount conventions. The abbreviations for such units as "metre", "second" and "gram" will now follow the number of units rather than precede it, so that braille will follow the standard print convention. The rules about fount indicators have been changed so that letters will now be assumed to be small Latin unless shown otherwise. Another important change is the extension of the use of the sign to code any sort of special function, e.g. lim, exp, det, etc., as well as the trigonometric and hyperbolic functions. At the same time some of the old word abbreviations such as for "rectangle", have been deleted.

A few alterations have been made in order to simplify the rules and increase conceptual clarity. For example, the numerator of a fraction will always be shown even when it is 1, so that $\frac{1}{2}$ will now be written  rather than . Similarly the exponent 2 will always be shown, so that x^2 will be written  rather than . It will no longer be necessary to decide whether an upper symbol in print should be treated as a superscript or as an exponent; the same superscript sign will now serve in both cases. The special method for dealing with limits of integrals has been deleted. Some new symbols have been added to the code to deal with certain print symbols which were not catered for before.

There was a strong feeling in the committee that the time had come to rewrite and restructure the code book. The two part structure of the old code has been abandoned, and it is hoped that the way the new edition is organized will make it much easier to use.

I would like to thank all the members of the committee for being so generous with their time and expertise and for making our discussions lively but good humoured. I am sure that all the other members will join me in expressing our appreciation of the work done by the Braille House staff in producing all the documents so efficiently in both braille and print.

A. W. Chatters

Bristol, November 1986

INTRODUCTION TO 2005 EDITION

The current edition of *Braille Mathematics Notation* contains some minor amendments to the 1987 edition prompted by the publication of the 2004 edition of *British Braille*. The main issues relate to the new rules on capital indicators. For conformity with *British Braille* the double capital indicator is now used for a sequence of two or more capital letters rather than for sequences of three or more. This change has been carried through to other fount indicators in this code, as well as to unit abbreviations. Also, single capital letters standing alone or starting a mathematical expression in ordinary text now require a letter sign before the capital sign, as in *British Braille*, thus removing a possible ambiguity with single letter wordsigns.

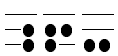
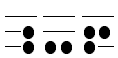
The oblique stroke has been changed to the 2-cell sign in line with *British Braille*. The force of the numeral sign no longer carries over the hyphen, again in line with *British Braille*.

Unit abbreviations starting with a lower case letter followed by a capital letter, such as mW, now require a letter sign before the m, which was previously not the case.

June 2005

TABLE OF BRAILLE MATHEMATICAL SIGNS

The braille signs in the following table should generally be used to represent the corresponding print signs whatever their meaning. In a few cases, however, more than one braille sign has the same print equivalent (e.g. the signs for $\cdot$, $<$): for these the correct braille sign should be used according to the meaning or use stated in brackets.

Sign	Meaning
	$+$ (plus)
	$-$ (minus)
	$\times$ (multiplied by, cross)
	$\div$ (divided by)
	$\pm$ (plus or minus)
	$\mp$ (minus or plus)
	/ (oblique stroke, fraction line sign)
	continued fraction symbol
	$\cdot$ (multiplication dot, and (in logic))
	$*$, $\circ$, etc. (an operation sign, e.g. $x * y$)
	$*$, $\circ$, etc. (a second operation sign)
	$\surd$ (root)
	$=$ (equals)
	$\equiv$ (equivalent to)
	$\sim$ (swung dash)

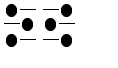
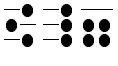
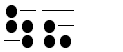
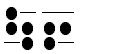
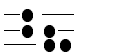
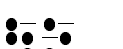
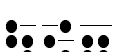
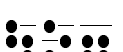
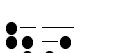
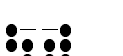
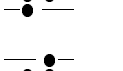
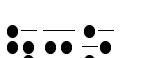
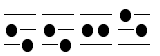
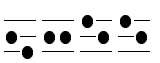
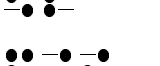
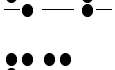
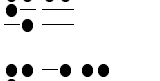
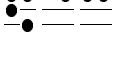
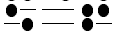
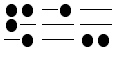
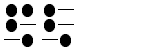
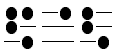
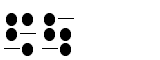
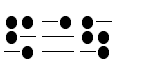
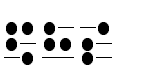
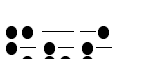
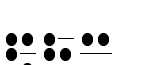
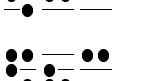
Sign	Meaning
	$\simeq$
	$\approx$, $\fallingdotseq$, or $\doteqdot$ (approximately equal to)
	$\cong$
	$\propto$ (varies as, proportional to)
	$\triangleq$, $=_{\mathrm{Def}}$ (is defined as)
	$\barwedge$ (is projective with)
	$<$ (less than)
	$\leq$ (less than or equal to)
	$>$ (greater than)
	$\geq$ (greater than or equal to)
	$\ll$ (much less than)
	$\gg$ (much greater than)
	$><$ (greater than or less than)
	$\lesssim$ (less than or approximately equal to)
	$\gtrsim$ (greater than or approximately equal to)
	numeral sign

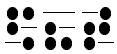
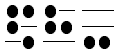
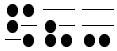
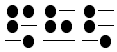
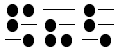
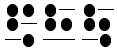
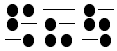
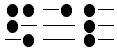
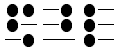
Table of Braille Mathematical Signs

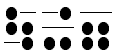
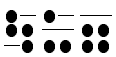
Sign	Meaning
	. (decimal point)
	, (comma or space marking thousands, etc.)
	˙ (recurring decimal sign)
	¯ (bar over negative number, e.g. $\bar{2}.345$)
	(
	)
	[
	]
	{
	}
	< (angle bracket)
	> (angle bracket)
	| (vertical bar: modulus, determinant, such that, divides, restricted to, evaluated at, etc.)
	‖ (double vertical bar: norm)
	matrix bracket
	∴ (therefore)
	∵ (since)

Sign	Meaning
	∩ (intersection)
	∪ (union)
	∧ (and, meet, vector product)
	∨ (or, join)
	\ (set difference)
	⊂ (contained in)
	⊃ (contains)
	⊆ (contained in or equal to)
	⊇ (contains or equal to)
	∈ (is an element of)
	∋ (reverse of ∈)
	¬ (not)
	& (ampersand, and)
	∀ (for all)
	∃ (there exists)
	∅ (empty set)
	⊢ (is a theorem)
	⊣ (reverse of ⊢)
	⊨ (is valid)
	⫤ (reverse of ⊨)

Sign	Meaning
	$\rightarrow$
	$\leftarrow$
	$\vec{\ }$ (superscript or subscript arrow indicating vector e.g. $\vec{AB}$)
	$\overleftarrow{\ }$ (superscript or subscript arrow indicating vector e.g. $\overleftarrow{AB}$)
	$\longrightarrow$ (arrow with long shaft)
	$\longleftarrow$ (arrow with long shaft)
	$\mapsto$
	$\rightarrowtail$
	$\leftarrowtail$
	$\twoheadrightarrow$
	$\twoheadleftarrow$
	$\Rightarrow$
	$\Leftarrow$
	$\leftrightarrow$
	$\Leftrightarrow$
	$\rightharpoonup$
	$\leftharpoonup$
	$\rightharpoondown$
	$\rightleftharpoons$, $\overrightarrow{\leftarrow}$

Sign	Meaning
	$\overrightarrow{=}$, $\overrightarrow{\leftarrow}$
	$\overrightarrow{=}$, $\overrightarrow{\leftarrow}$
	$\uparrow$, $\nearrow$
	$\downarrow$, $\searrow$
	sin
	$\sin^{-1}$, arcsin
	cos
	$\cos^{-1}$, arccos
	tan
	$\tan^{-1}$, arctan
	sec
	$\sec^{-1}$, arcsec
	cosec
	cosec^{-1}, arccosec
	cot
	$\cot^{-1}$, arccot
	sinh
	$\sinh^{-1}$, arcsinh
	cosh
	$\cosh^{-1}$, arccosh
	tanh

Sign	Meaning
	$\tanh^{-1}$, arctanh
	sech
	sech^{-1}, arcsech
	cosech
	cosech^{-1}, arccosech
	coth
	$\coth^{-1}$, arccoth
	log
	$\log^{-1}$, antilog
	colog
	grad (gradient)
	curl or rot
	div (divergence)
	∇ (nabla, del)
	$\angle$, $\hat{}$ (angle)
	$\bigcirc$ (circle symbol)
	$\triangle$ (triangle symbol, symmetric difference)
	$\square$ (square symbol, vector or wave operator)
	$\triangleleft$ (is a normal subgroup of)
	$\triangleright$ (reverse of $\triangleleft$)

Sign	Meaning
	$\trianglelefteq$ (is a normal subgroup of or equal to)
	$\trianglerighteq$ (reverse of $\trianglelefteq$)
	$\parallel$ (parallel symbol)
	$\perp$ (perpendicular symbol)
	$\int$ (integral)
	$\oint$ (closed line integral)
	∞ (infinity)
	■, □, //, etc. (end of proof)
	# (hash)
	! (factorial)
	. (point notation of partial correlation or regression in statistics, e.g. $x_{12.3}$)
	$\coprod$ (coproduct)
	∂ (curly d, partial derivative d)
	$'$ (dash, prime, e.g. x')
	* (star, e.g. x^*)
	$\bar{}$ (bar, e.g. $\bar{x}$)
	$\hat{}$ (hat, e.g. $\hat{x}$)
	$\check{}$ (inverted hat, e.g. $\check{x}$)

Sign	Meaning
	˜ (tilde, e.g. $\tilde{x}$)
	† (dagger, e.g. $x^{\dagger}$)
	⌐ (actuarial symbol: years, e.g. $\overline{x}$)
	˙ (differentiation dot, e.g. $\dot{x}$)
	¨ (double differentiation dots, e.g. $\ddot{x}$)
	‴ (triple differentiation dots, e.g. $\dddot{x}$)
	° (degrees)
	′ (minutes)
	″ (seconds)
	^{c} (radians)
	Å (ångström)
	£ (pound sterling)
	$ (dollar)
	¢ (cent)
	€ (euro)
	% (percent)
	unit separation sign
	mathematical hyphen
	separation sign

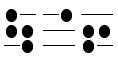
Sign	Meaning
	index separation sign
	differential operator separation sign
	superscript sign
	subscript sign
	top index sign
	bottom index sign
	index termination sign
	matrix line indicator
	'word' indicator
	small Latin letter sign
	capital Latin letter sign
	small bold Latin letter sign
	capital bold Latin letter sign
	small Greek letter sign
	capital Greek letter sign
	double capital Greek letter sign
	small letter sign
	capital letter sign
	small letter sign
	capital letter sign

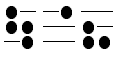
Table of Braille Mathematical Signs

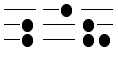
Sign	**Meaning**
	small letter sign
	capital letter sign
	small letter sign
	capital letter sign
	crossed symbol sign

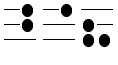
Modification of Signs

(a) When it is required to show that a non-alphabetic symbol is printed in bold or large type as distinct from the usual print sign, a dot 4 is inserted after the , or for compound signs with these prefixes.

Ex.1 $\bigcup$ 

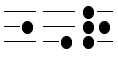
$\bigcap$ 

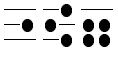
$\times$ 

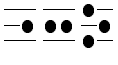
$\bigwedge$ 

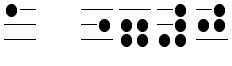
This indication is not made when it is just a style of printing and of no mathematical significance. It is not usually shown for bold ∇ or $\Box$ signs in braille.

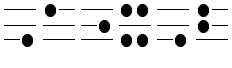
(b) A symbol consisting of a struck out letter or sign is coded by inserting a dot 5: this follows the for signs prefixed by , replaces the for signs prefixed by , and precedes all other signs (i.e. precedes a letter fount sign if present). If a dot 5 is already present at this point, it must be retained.

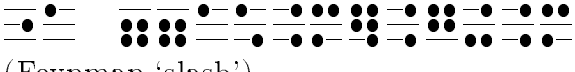
Ex.2 $\not\mathbb{R}$ 

$\not\leq$ 

$\not\to$ 

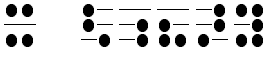
$a \neq 0$ 

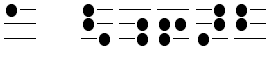
$A \not= B$ 

$\not{a} \equiv a_\mu \gamma^\mu$ 
(Feynman 'slash')

Analogously, $\not> \not\geq$ is coded as , and $\not\supset \not\geq$ as , etc.

(c) Symbols printed in a circle or a square may be coded by placing the sign in round or square brackets respectively, with the compound sign obeying the usual rules for the enclosed sign.

Ex.3 $x \otimes y$ 

$a \oplus b$ 

BRAILLE MATHEMATICS NOTATION

1 GENERAL REMARKS

1.1 Set out mathematical expressions are generally brailled beginning in cell 5 with runovers in cell 7, whatever the setting in print. (See §**13** for further details concerning layout of mathematical text.)

1.2 Dot 6 precedes any punctuation sign except a hyphen or dash, used within or immediately following a mathematical expression. (See for example §**3.4**, **6.2** and **9.1.1**, but note the examples in §**7**, and §**14.9** for unit abbreviations.)

1.3 Dot 5 is used as a mathematical hyphen when it is necessary to divide a mathematical expression at the end of a braille line, whether or not a division is made at that point in the print.

The following rules for dividing mathematical expressions should be observed:

(a) Do not separate letters from their letter signs or numbers from their numeral signs.

(b) Do not divide an expression so as to separate indices or dashes, stars, hats, etc. from their terms, or functions from their arguments (unless the function is unusually long).

(c) Do not divide immediately **after** a sign which is normally spaced before but not after (e.g. $+$, $=$, $\equiv$, $\leq$, $\subset$, $\sim$, $\rightarrow$, etc. (see §**3.2**)); or after the signs for $\cdot$, for /, or an opening bracket.

(d) Do not divide immediately **before** a sign, or before a closing bracket.

(e) Do not divide an expression at a point where a space is specially omitted to unite it (e.g. in indices (§**6.5**) or coordinates (§**9.1.2**)).

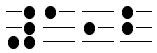
It is generally not good practice to divide a short expression (e.g. $x = 1$) which could be conveniently brailled complete on a new line.

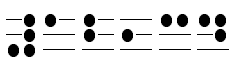
1.4 If the print text contains symbols which are not listed or otherwise covered in this code, special braille signs may be devised to represent them, or else the meaning of existing signs may be changed for the purpose. Any such non-standard notation should be stated beforehand (e.g. at the beginning of the work in which it appears, or at the point of use). (See in particular §**2.2.4**.)

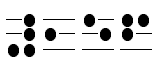
2 NUMERAL AND LETTER SIGNS

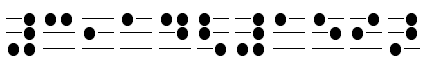
2.1 The numeral sign indicates that immediately following letters *a*-*j* represent numerals 1-9 and 0 until the sequence is ended by a space, a letter *k*-*z*, or any other sign except those given in §**2.1.1** – **2.1.5** below.

2.1.1 The decimal point is coded as dot 2.

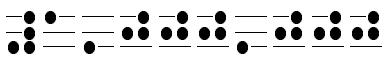
Ex.1 1.2 

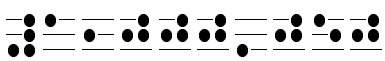
12.34 

.56 

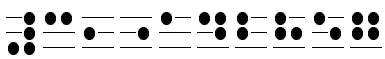
3.14(159) 

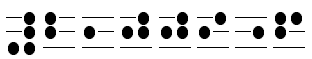
2.1.2 The print comma or space separating groups of digits such as thousands or thousandths is coded as dot 3.

Ex.2 1,000,000 

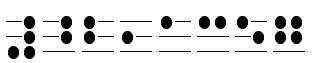
1.000 050 

2.1.3 Recurring decimals are coded by inserting a dot 5 before the recurring sequence, that sequence being indicated in print by dots placed above its first and last digits (or its single digit).

Ex.3 $3.\dot{1}42857\dot{1}$ 

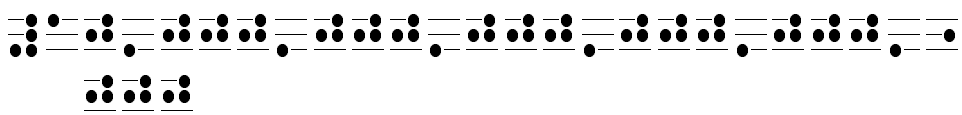
$2.0096\dot{}$ 

2.1.4 A bar indicating a negative part of a number (a notation used for logarithms) is coded by inserting the sign between the numeral sign and the number.

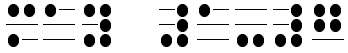
Ex.4 $\bar{2}.1357$ 

2.1.5 A number should only be divided (using the mathematical hyphen, dot 5) if it is too long for a whole line. In such cases the force of the numeral sign carries over the dot 5 and is not reasserted at the beginning of the new line.

Ex.5 10,000,000,000,000,000,000



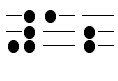
2.1.6 The force of the numeral sign does not carry over the literary hyphen .

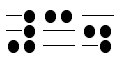
Ex.6 May 1-7 

1.2-2.3

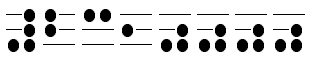
2.1.7 Numeral signs may be omitted altogether from tables or worked calculations to save space or to allow an uncluttered presentation: in such cases advance notice should be given.

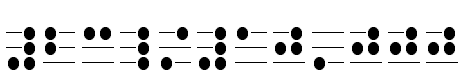
2.1.8 Simple numerical fractions are coded by stating the denominator as a lower number following the numerator in the upper position.

Ex.7 $\frac{1}{2}$ 

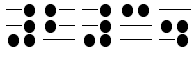
$\frac{3}{4}$ 

If either the numerator or denominator has a comma or space in the print indicating thousands, etc., the comma may be omitted in the braille. Alternatively, if the comma is to be retained, an oblique stroke and a second numeral sign should be inserted, with the denominator written in the upper position. (This latter method retaining the oblique stroke may also be used for fractions without the separating dot 3's when it is considered desirable to emphasize the fraction line (e.g. in calculations).)

Ex.8 $\frac{23}{10,000}$ 

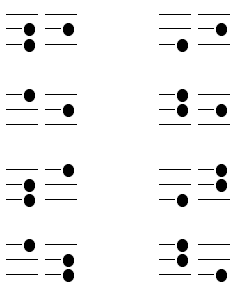
or 

2.1.9 In mixed numbers the numeral sign is repeated for the fractional part.

Ex.9 $2\frac{3}{4}$ 

2.2 A letter fount sign gives the fount of the immediately following letter only:

	Small	**Capital**
Latin		
Greek		
Bold (or underlined) Latin (See §**2.2.5**)		
Bold Greek		
Bold numeral (two kinds rather than small and capital)		

	Small	**Capital**
Other founts (See §**2.2.4**)		

2.2.1 The small Latin letter sign is generally omitted, all letters being assumed to be small Latin unless shown otherwise. It must, however, be stated when the letter

(a) Stands alone or begins a mathematical expression in ordinary text;

(b) Is an *a*-*j* immediately following a number (but not a lower number, e.g. the denominator of a simple fraction), even at the beginning of a line;

(c) Immediately follows a sequence of letters brailled with a double letter fount sign (§**2.2.3**);

(d) Immediately follows a vertical bar | ( );

(e) Is an *o* within brace brackets { };

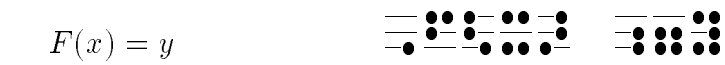
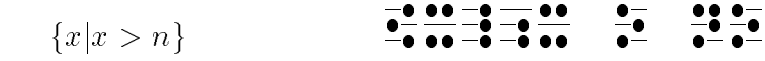
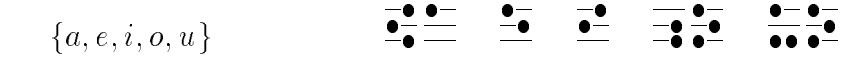
(f) Is an *a* immediately after an opening square bracket ( ) in ordinary text;

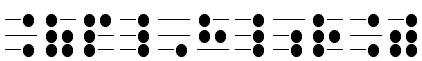
(g) Is the first letter of a small Roman numeral, or a capital Roman numeral coded as such (§**2.3**);

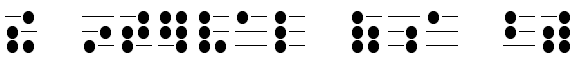
(h) Immediately follows a prefixed function or expression (see §**7.1**).

(See §**14.2** for unit abbreviations.)

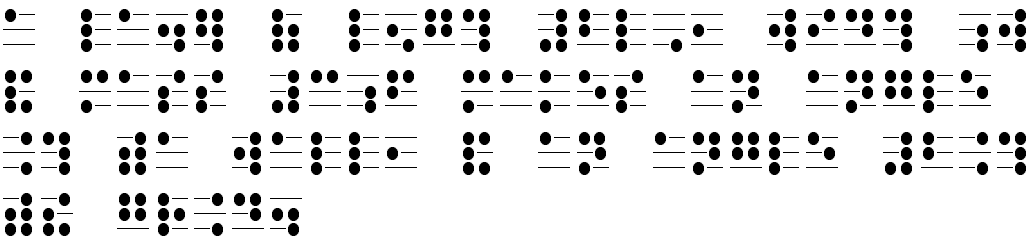
Ex.10

x	
$F(x) = y$	
$5c$	
$\frac{3}{4}c$	
π	
$2\pi r$	
Ax	
$\{x \mid x > n\}$	
$\{a, e, i, o, u\}$	

$\langle p|H|r\rangle$ 

The interval $[a, b]$ 

Ex.11 A ladder of length $2l$, width w and mass $3f$ makes an angle θ with a wall, and an angle 2θ with the ground.

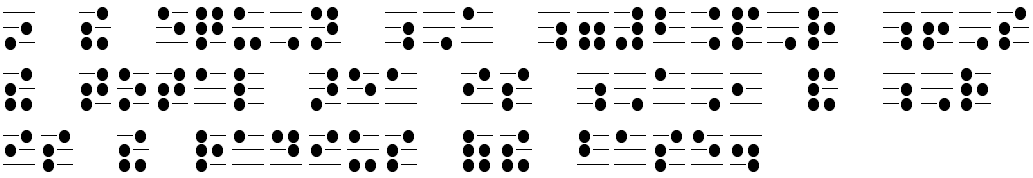


2.2.2 The sign is also used before a single capital Latin letter, possibly followed by a simple numerical subscript, when the letter

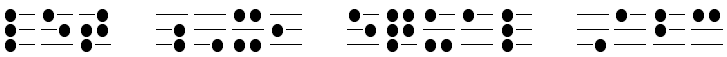
(a) Stands alone within ordinary text;

(b) Begins a mathematical expression in ordinary text and is followed by a space.

Ex.12 In the equation $A = 2\pi R + S$ the total area is A, and R is the radius of the base.

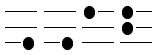


Let X_1 equal Abc

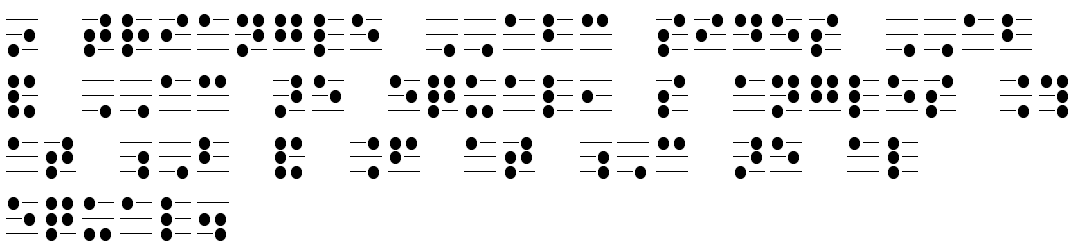


(Note that in accordance with the above, a capital Latin letter acting as a label in a mathematical diagram and not adjacent to ordinary words does not require a letter sign.)

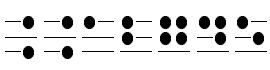
2.2.3 If two or more letters of the same fount other than small Latin (or Roman numbers treated as such (§**2.3**)) occur in sequence, they are preceded by the appropriate double letter fount sign which has force until the sequence is ended by a space or any other sign except a dash, star, or numerical subscript. To avoid misreading (as bar), the Greek letter η (small or capital) should always be preceded by the single letter fount sign, and therefore terminates a double Greek letter fount sign sequence.

Ex.13 AB 

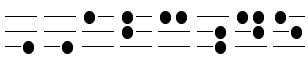
In triangle ABC sides AB and AC are equal, so angles θ at B and ϕ at C are also equal.



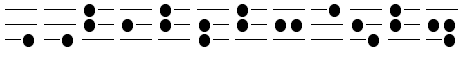
$\alpha\beta\gamma\delta\epsilon$



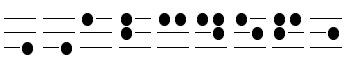
$ABCde$

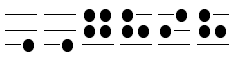


$B_1B_2B_3^*B_4$

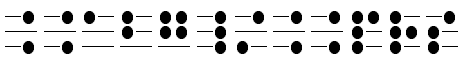


$ABCDEFGHIJ$

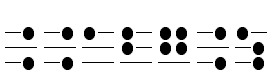




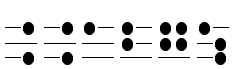
$\dfrac{\alpha\beta\gamma}{\pi\rho\sigma}$



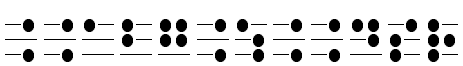
$\alpha\beta\gamma\eta$



$\alpha\beta\bar{\gamma}$

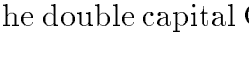


$\alpha\beta\gamma\eta\delta\sigma\rho$

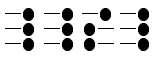


Exceptionally is used as the double capital Greek letter fount sign to distinguish it from double bars.

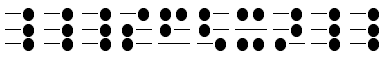
Ex.14 $\Gamma\Delta\Pi\Sigma$



$|\Sigma|$



$\|\Sigma f(x)\|$

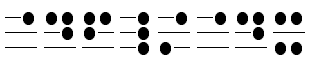


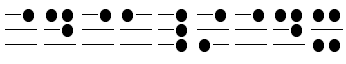
2.2.4 Undesignated letter fount signs are used for other founts according to the particular requirements of the text. When they are used their meaning should be stated. If the fount is particularly predominant and frequent, it may in some cases be advantageous to adopt a single cell letter fount sign for the purpose, to avoid over use of a two cell fount sign. Such non-standard use should be explained.

2.2.5 Letters printed or written with an underline or under-tilde used as a fount sign (e.g. $\underline{a}$, $\underset{\sim}{U}$), may generally be coded with the dot 4 or dots 45 letter fount signs without further

comment, unless bold letters also occur in the text, or if the underline or tilde needs special or individual indication.

Dot 4 is also used to show the bold fount of other signs (see paragraph (a), P.7), and is the standard fount sign used to indicate ∂ (curly d) in partial derivatives, etc. without further comment.

Ex.15 $\dfrac{\partial f}{\partial x}$ 

$\dfrac{\partial \mathbf{a}}{\partial x}$ 

2.2.6 A special script l (ℓ) is sometimes used in print to distinguish the letter from number 1. This is unnecessary in braille: code the letter as an ordinary letter l.

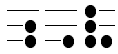
2.2.7 $\sum$ and $\prod$ indicating summation and product are conventionally normally coded as and respectively, whether or not written as bold letters in print.

2.2.8 $\aleph$ (aleph) and $\wp$ (an unusual letter π denoting the Weierstrassian elliptic function) are coded as letters a and p respectively with suitable letter fount signs. This notation should be explained when first used.

2.2.9 The Dirac (or Planck) constant symbol $\hbar$ used in physics should be coded as $\bar{h}$ unless this notation would be ambiguous in the particular context. Otherwise it may be sufficient to code it simply as h if that letter is not otherwise used, or else a particular letter fount sign should be adopted for the purpose: in both these latter cases the notation should be explained when first used.

2.3 Roman numerals are normally written as in literary braille; if small, or brailled as such (i.e. in non-capitalized braille), they are preceded by the dots 56 letter sign. If, however, they are indistinguishable in the print from ordinary algebraic letters, they may be coded as such according to the rules in the rest of this section.

Note that even in non-capitalized braille it may be necessary to show that Roman numerals are capital.

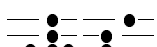
Ex.16 V 

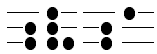
(non-capitalized braille)

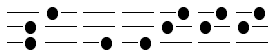
II

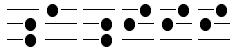
(non-capitalized braille)

iii

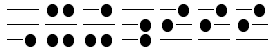
$\mathrm{V}a$ 

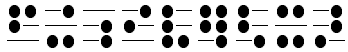
 (non-capitalized braille)

$a\mathrm{III}$ 

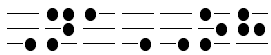
 (non-capitalized braille)

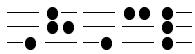
X^{III}

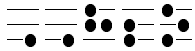
 (non-capitalized braille)

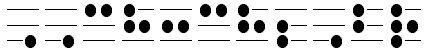
$f^{\mathrm{iv}}(x)$ 

2.4 Special letter fount rules exist for brailling chemical formulae; for this reference should be made to *Braille Science Notation*. However, it is possible to represent such formulae unambiguously using the conventions of mathematics braille. In such cases it is advisable to avoid using the dots 56 sign between the letters of a two-letter element symbol by instead using a dot 6 sign before the first letter in the symbol.

Ex.17 NaOH 

HCl 

$\mathrm{H_2O}$ 

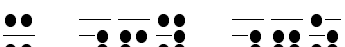
$\mathrm{CH_3CH_2Br}$ 

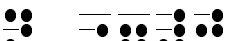
Refer also to *British Braille* 6.5.7 where examples are given of such formulae brailled using literary conventions only, in which the subscript sign is retained.

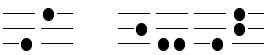
3 THE SPACING OF BRAILLE SIGNS

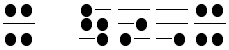
3.1 The following gives general rules for the spacing of braille signs within a mathematical expression. Spaces are, however, omitted when these signs are used in indices or coordinates, etc.: these cases are dealt with in the particular sections (see for example §**6**, **9**, **10**). An initial space is also omitted when the sign immediately follows an opening bracket, and similarly a following space is omitted when the sign immediately precedes a closing bracket.

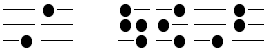
3.2 Operation and relation signs, etc. prefixed by , or (except for $\cdot$ and for $\emptyset$); all arrows; the signs for $\leq$, $\geq$, $\equiv$, $\approx$ $\triangleq$ or $\doteq$, $\simeq$, $\cong$, $\gtrsim$, $\lesssim$, $\propto$, $\sim$, $\parallel$ (parallel to), $\perp$; and the negation of these signs, are spaced before but not after.

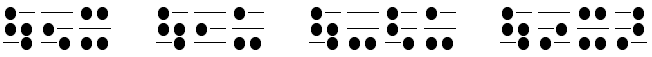
Ex.1 $x + y = z$ 

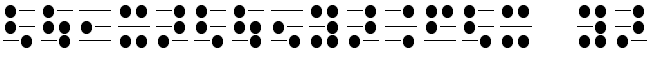
$n \neq 0$ 

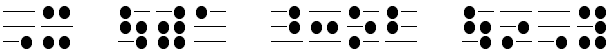
$A \sim B$ 

$x \in X$ 

$A \subset B$ 

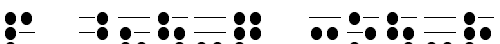
$\exists x \forall u \neg(u \in x)$ 

$(\forall x)(\exists y)\phi(x, y)$ 

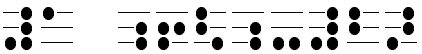
$X \ni a \mapsto b \in Y$ 

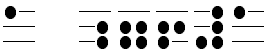
3.2.1 The sign for ¬ (not) may be immediately preceded by a sign which would normally be unspaced after (e.g. →, ·, ∨, &, etc.). In such cases, ¬ should be brailled unspaced. The same also applies to the signs for ∼, +, −, etc.; however, when two signs beginning with occur in succession, the is omitted before the second sign.

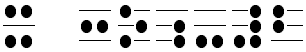
(Exceptionally, brackets should be used for the juxtaposition of + and − signs, as means ± etc. See §**5.3**.)

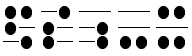
Ex.2 $p \wedge \neg q \rightarrow \neg r$ 

$p \vee \sim p$

$1 + -2$ 

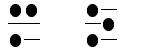
$a = +1$ 

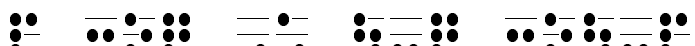
$x \rightarrow -2$ 

$\sin -x$ 

3.2.2 Arrows between ordinary words (e.g. ⇒ 'implies') should remain spaced.

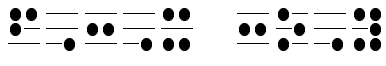
3.3 The signs for >, ≫, <, ≪, ><, ∴, ∵, ⊢, ⊨, ⊣, ⫤, ■, and the negation of these signs are spaced on both sides.

Ex.3 $m > n$ 

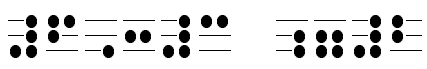
$p \rightarrow q \therefore \neg q \rightarrow \neg p$ 

3.4 Punctuation is generally spaced according to print, though it should not usually be left spaced both before and after in braille.

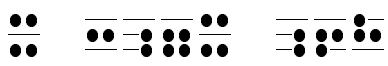
Ex.4 $f : X \to Y$ or
$f \ : \ X \to Y$



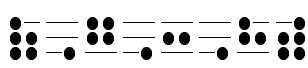
$6 : 3 = 2$



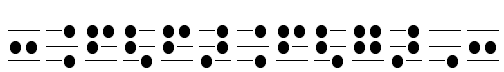
$x := x + h$



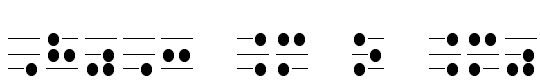
$[G : H]$



$: \phi(f)\phi(g) :$

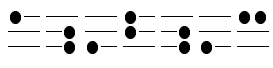


$H_0 : \mu > \mu_0$

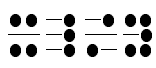


3.5 Other signs are freely brailled unspaced, subject to satisfying the constraints on adjacent signs given by the above rules (§**3.2**–**3.4**). See, however, §**7.2** for the treatment of adjacent word segments representing special functions, etc.

Ex.5 $a \cdot b \cdot c$

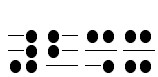


x/y

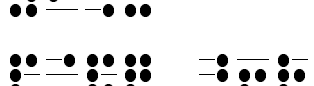


$\sqrt{x}$

$2\sqrt{x}$

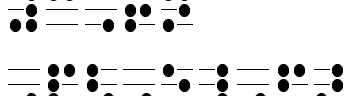


$p \& q \vee r$

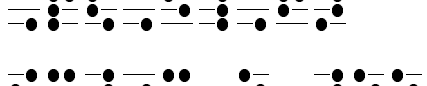


$3!n$

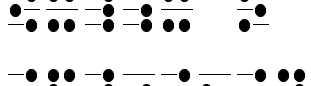
$P(E|F)$



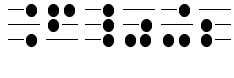
$\{x|x > 5\}$



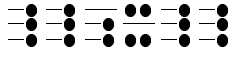
$\phi \nabla^2 \psi$



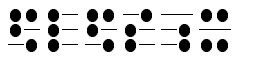
$\|x\|$



$\log \sin x$



$2t \sin \theta$

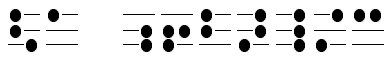


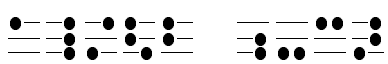
(Refer also to the various examples throughout this code.)

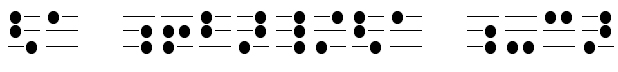
4 THE OBLIQUE STROKE AND FRACTION LINE

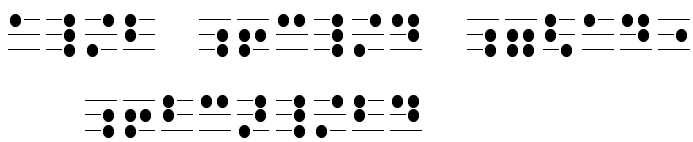
4.1 The sign is used in braille to represent both the oblique stroke and the horizontal fraction line in print (except where it is omitted for simple numerical fractions (§**2.1.8**, **2.1.9**)). When it represents the horizontal fraction line, brackets must be inserted in braille in the following cases:

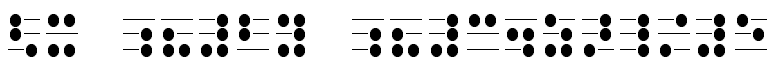
(a) To unite the numerator or the denominator of a fraction, consisting of two or more spaced terms or expressions in braille, or which is itself a fraction brailled with the sign.

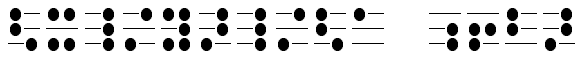
Ex.1 $\dfrac{a+b}{c}$ 

$\dfrac{a}{b-c}$ 

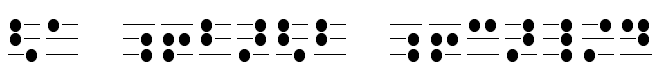
$\dfrac{a+b}{a-c}$ 

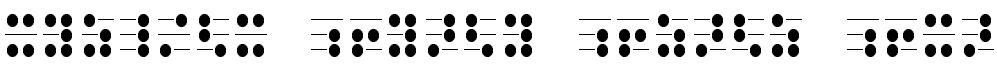
$\dfrac{a}{b}+\dfrac{c}{d}=\dfrac{ad+bc}{bd}$ 

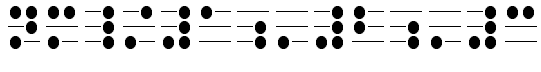
$\dfrac{x\times 2y\times \frac{3}{4}z}{5}$ 

$\dfrac{\frac{x}{y}}{a+b}$ or $\dfrac{x/y}{a+b}$ 

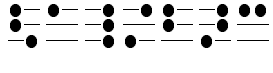
(If the numerator or denominator consists only of terms multiplied together and not separated by spaces, extra brackets are not required.)

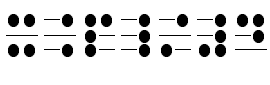
Ex.2 $\dfrac{(a+b)(b+c)}{d}$ 

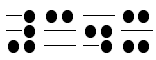
$\dfrac{xyz}{(x+y)(y+z)(z+x)}$ 

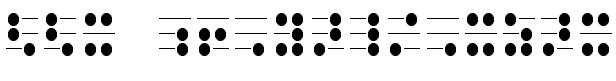
$\dfrac{nm}{1\cdot 2\cdot 3}$ 

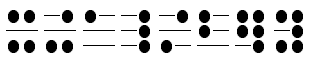
(b) To separate a fraction brailled with an oblique stroke from a following unspaced multiplying term, unless the fraction is an index and thus terminated by a sign (see §**6.1**).

Ex.3 $\dfrac{a}{b}c$ 

$x\frac{\pi}{4}$ 

$\frac{3}{4}x$ 

$\frac{x+Y}{Xz}x$ 

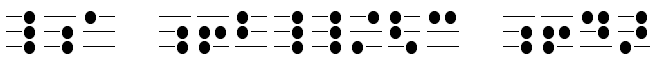
$x^{a/b}y$ 

5 BRACKETS

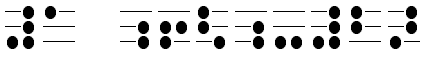
Rules concerning brackets occur in various sections of the code (e.g. §**4** The Oblique Stroke and Fraction Line; §**6** Indices; §**9** Coordinates and Sets; §**10** Matrices). The following should also be observed.

5.1 All brackets in print should be represented by brackets of the same type in braille.

5.2 Modulus bars serve as brackets.

Ex.1 $\frac{|a+b|}{c+d}$ 

5.3 Brackets should be inserted to enclose a signed number or term when preceded by a + or − sign.

Ex.2 $1+-2$ or $1+{}^{-}2$ 

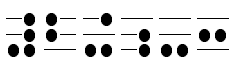
5.4 Insertion of brackets may be desirable in the braille to add clarity, even when not specifically required by the rules.

6 INDICES

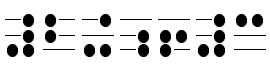
6.1 The superscript sign , indicates that the expression which follows is printed in the superscript position. Unsigned or negative simple whole number superscripts are written in the lower part of the cell without a numeral sign; except for this case, superscript expressions must be terminated by a sign if not by a space or a closing bracket enclosing the whole term.

Arrows such as →, ↑ or ↓ immediately following the superscript sign should be preceded by the dot 4 separator to avoid misreading of the cell.

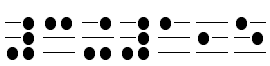
Ex.1 2^2

2^{-3} 

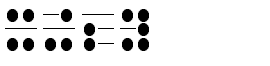
2^{+3}



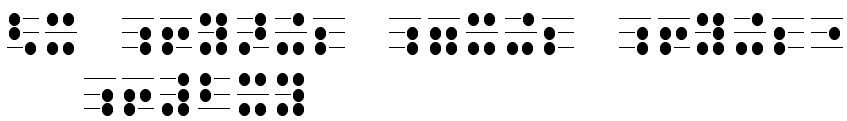
$3^{1.5}$



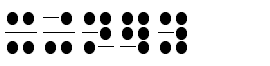
x^2y



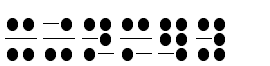
$(x+y)^2 = x^2 + y^2 + 2xy$



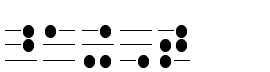
$x^{n}y$



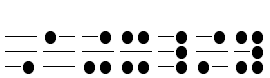
$x^{nm}y$



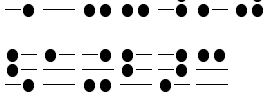
$\underline{A}^T$



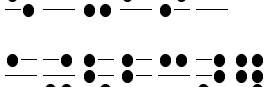
$A\frac{x}{y}B$



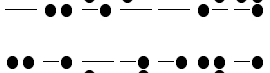
$(a^b)c$



$a^{(bc)}d$

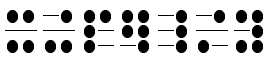


$\frac{x^2}{y^3}$

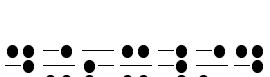


$\frac{x^p}{y^q} + \frac{a^p}{b^q}$

$\frac{d^2x}{dy^2}$

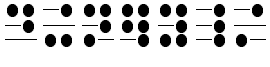


$\frac{d^ny}{dt^n}$

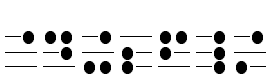


$\frac{\partial^2 f}{\partial x \partial y}$

$\overrightarrow{a}$

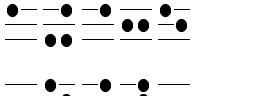


$E^{\perp}$



$\mathbf{A}^{\parallel}$

$X \xrightarrow{f} Y$

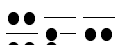


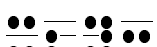
6.2 The subscript sign ⠩ , indicates that the expression which follows is printed in the subscript position. Unsigned or negative simple whole number subscripts are written in the lower part of the cell without a numeral sign, and when such indices follow letters, closing brackets or the integral sign the subscript sign is also omitted; except for these cases subscript expressions must be terminated by a ⠱ sign if not by a space or a closing bracket enclosing the whole term. As in print, x_{11} may represent x-one-one or x-eleven, etc.

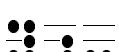
Arrows such as →, ↑ or ↓ immediately following the subscript sign should be preceded by the dot 4 separator to avoid misreading of the ⠒ cell.

Ex.2

x_2

x_{23} 

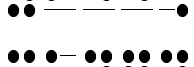
$x_2 y_3$ 

y_{-1} 

x_{+1}

$z_{0.5}$

245_8

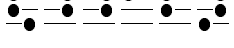
$x_n y$ 

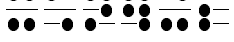
X_{n1}

x_{2n}

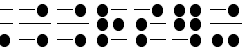
$(x_i)b$

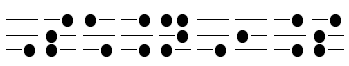
$(\mathbf{AB})_{11}$

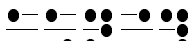
$x_n^{\,2}$ 

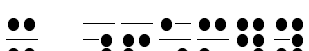
$x_n^{\,m}$ 

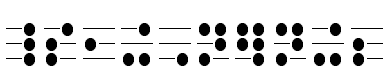
$T^{\mu\nu}_{\rho\sigma} S^{\sigma}_{\nu}$

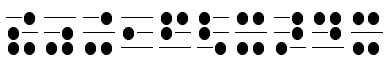
$T^{\mu}_{\rho;\sigma}$ 

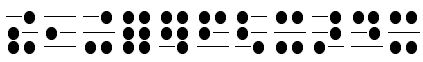
$S_{\nu,\sigma}$ 

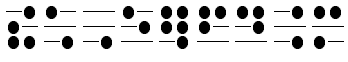
$a_{\overline{n}|}$ 

$x +_m y$ 

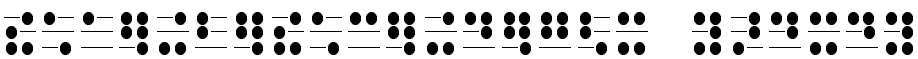
$\sum_{1}^{N} n^2$ 

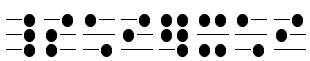
$\int_0^1 f(x)dx$ 

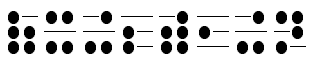
$\int_1^\infty f(x)dx$ 

$\int_E f d\mu$ 

$\int_a^b \int_c^d g(x,y)dxdy$



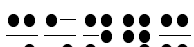
$\sum_i x_i$ 

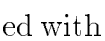
$[x^2]_1^n$ 

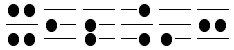
6.2.1 Root exponents are coded as lower (right-hand) indices.

Ex.3 $\sqrt{x}$

$\sqrt[3]{x}$

$\sqrt[n]{x}$ 

6.3 Lower numbers are used with the sign  representing the point in partial regression coefficients and partial correlations etc. in statistics, but are not used within indices in conjunction with any other sign except the minus sign as in §**6.1** and §**6.2** above.

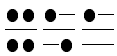
Ex.4 $x_{12\cdot 3}$ 

6.4 Expressions with several levels of indexation are coded by asserting superscript or subscript signs as required, remembering that a superscript or subscript sign does not cancel the force of a preceding such sign, but indicates a deeper level.

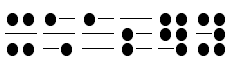
The index terminator only cancels 1 level of indexation, i.e. the last uncancelled superscript or subscript sign (lower numbers act so as to cancel immediately preceding

superscript or subscript signs without a ⠘). If more than one level needs to be cancelled at a point in an expression, the appropriate number of ⠘ signs should be used.

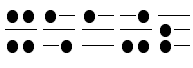
Ex.5 x_{a_1}



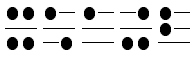
$x_{a_2}y$



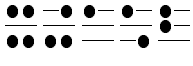
x_{a^2}



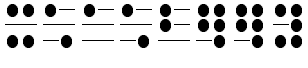
x_{a^b}



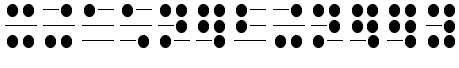
x^{a_b}



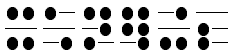
$x_{a_b}y$



$x^{a_n b^n}y$

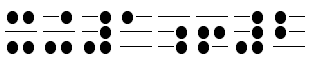


x_n^2

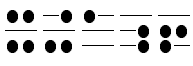


6.5 Indices consisting of two or more terms etc. which would normally be spaced in braille (see §**3**), should be united by omitting the spaces between those terms (if not already united by being enclosed in brackets). In such cases, a dot 4 separator should be inserted before signs such as $\rightarrow$, $\uparrow$, $\downarrow$, $\supset$, $\supseteq$ for which ambiguities could otherwise arise. As exceptions, the sign for $\geq$ and the signs which are normally spaced (e.g. $<$, $>$) should retain their normal spacing so that brackets enclosing the index are necessary in braille. Spaces are also retained in sequences of word segments denoting functions, etc. (see §**7.2**), so brackets are again required. (Subscript coordinates or sets enclosed in brackets are coded as in §**9.1**.)

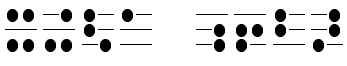
Ex.6 x^{1+2}



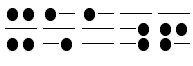
x^{a+b}



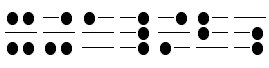
$x^{(a+b)}$



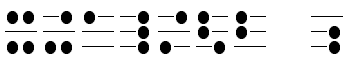
x_{a+b}



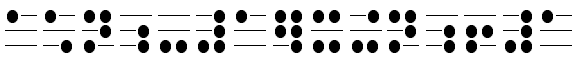
$x^{\frac{a}{b}+c}$



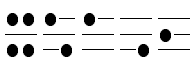
$x^{\frac{a}{b+c}}$



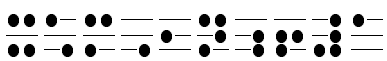
$a_{n-1}x^{n+1}$



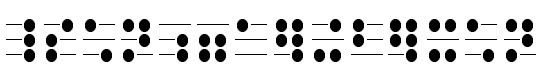
$x_{a,b}$



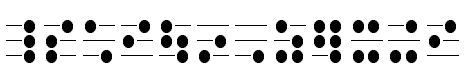
$x_{m,n+1}$



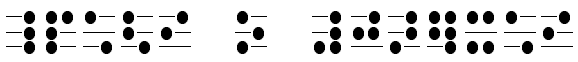
$\sum_{n=a}^{b} x_n$



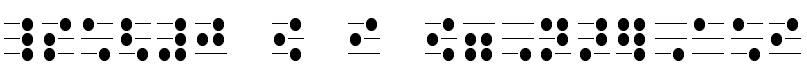
$\sum_{i \in Z} x^i$



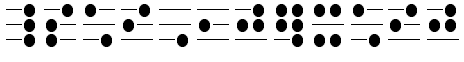
$\prod_{i>0} x_i$



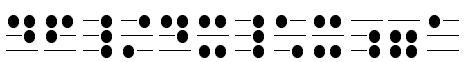
$\sum_{0<i\leq N} A_i$



$\sum_{i,j} x_{ij}$

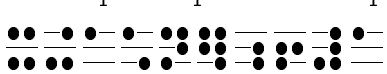


$\left.\frac{df}{dx}\right|_{x=a}$

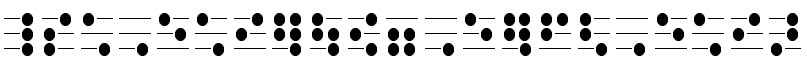


6.5.1 When omitting spaces it will be necessary in some cases to insert the sign to terminate an index within the superscript or subscript.

Ex.7 x^{a_n+1}



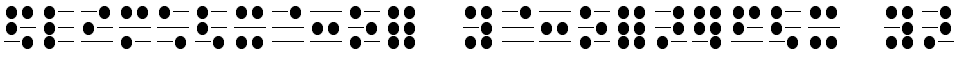
$\sum_{E_i \subseteq E} f(E_i)$



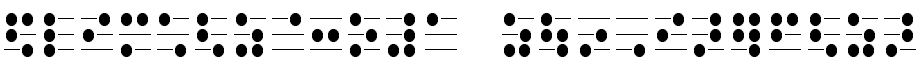
6.5.2 Brackets should be used to enclose an index consisting of two or more separate expressions.

Ex.8 $\sum_{\substack{i\geq 0 \\ j\geq 1}} x_{ij}$

$\lim_{\substack{x\to\infty \\ y\to\infty}} f(x,y)$

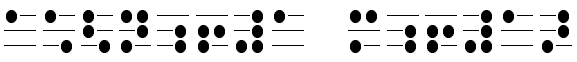


$\lim_{z\to 1, z\in I} f(z)$

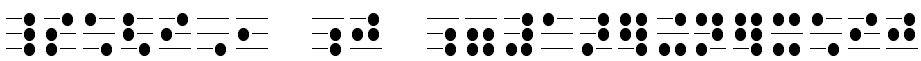


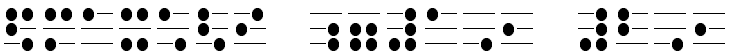
(As shown in the last example, when brackets are used to enclose the index and the individual expressions are brailled unspaced, a separating comma may be omitted as is done with coordinates, etc. (§**9**).)

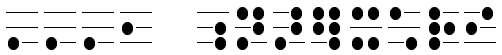
6.5.3 Brackets (rather than the omission of spaces) should be used to unite more complex indices and those in which the spacing needs to be retained, as well as to unite indices which are to be divided at the end of a braille line.

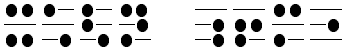
Ex.9 $a_{n+1\ m+1}$ 

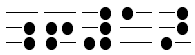
$\sum_{i,j=1}^{n} x_{ij}$



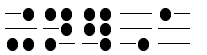
$\max_{i=1,2,\ldots,n} x_{ri}$ 

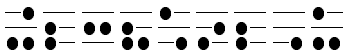


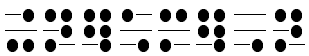
x_{n+m+1} 

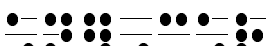


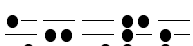
6.6 Indices on the left are preceded by the appropriate or signs and followed, if necessary, by signs.

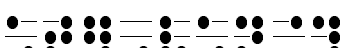
Ex.10 ${}^{n}A$ 

${}^{238}_{92}U$ 

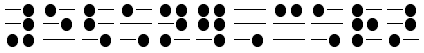
${}^{n}_{m}N$ 

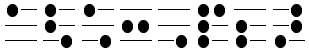
${}_{n}C_{r}$ 

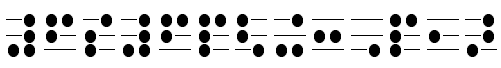
${}_{3}P_{2}$ 

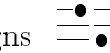
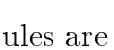
${}_{t}V_{\overline{n}|}$ 

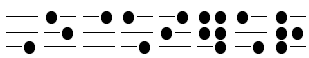
6.6.1 Additional brackets may be required in braille to make clear the proper attachment of such indices in an unspaced sequence.

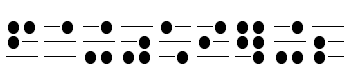
Ex.11 $5\ {}_{n}C_{r}$ 

$a\ {}_{3}P_{2}$ 

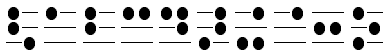
$6s\,6p\,{}^3P_1$ 

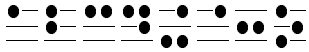
6.7 The signs  and  are used to indicate indices directly above or below a term respectively, when the distinction between these and standard indices is necessary. The same rules are employed as for the and index signs; if a combination of such signs occurs in an expression, each must be cancelled separately.

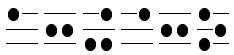
Ex.12 $\underset{i}{E}\,r$ 

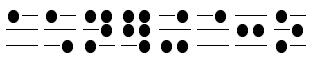
$\mathring{f}_i^{\,2}$ 

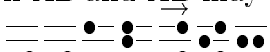
6.8 When a superscript or subscript arrow extends over several signs, that extent may generally be indicated in braille by using brackets. (Lower numbers are not regarded as full signs here, but as being part of the sign to which they are an index, so that cases such as $\overrightarrow{a_3}$ and $\overrightarrow{a^2}$ may be coded without brackets. Special functions (see §**7**) are also regarded as single entities in this connection.)

Ex.13 $\overrightarrow{abcd}$ 

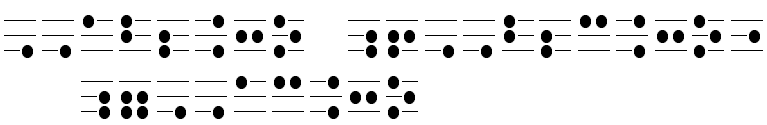
$abc\vec{d}$ 

$\overrightarrow{a_3}$ 

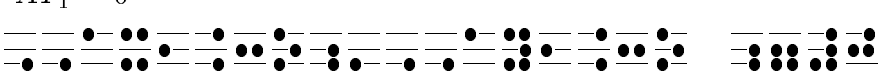
$\overrightarrow{a_n}$  (brackets unnecessary)

In the particular case of vectors or directed line segments (determined by two points), both $\overrightarrow{AB}$ and $A\underset{\rightarrow}{B}$ may be compactly coded as , and both $\overleftarrow{AB}$ and $\underset{\leftarrow}{AB}$ as  , etc.

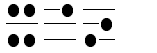
Ex.14 $\overrightarrow{AB}_2 + \overrightarrow{B_2C} = \overrightarrow{AC}$

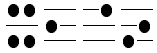


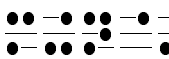
$\overrightarrow{AX}_1 \cdot \overrightarrow{AY}_1 = 0$

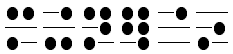


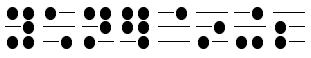
6.9 If a dash, star, hat, tilde, dagger, etc. is in its usual superscript position in print, it is brailled without an index sign.

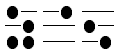
Ex.15 x' 

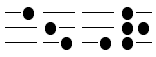
$x_1{}'$ 

$m^{n'}$ 

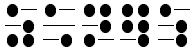
$m^{n\prime}$ 

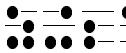
$y_n'^2$ 

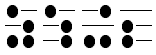
z^* 

$^*R$ 

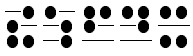
$\bar{z}$

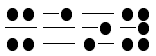
$\overline{z_n}$ 

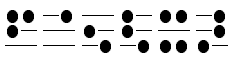
$\overline{z^2}$ 

$\bar{z}^2$ 

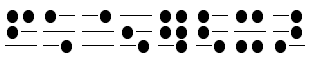
The index terminator is not required for such signs.

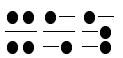
Ex.16 $\bar{\int} f dx$ 

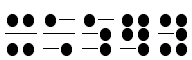
$x'y$ 

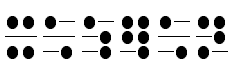
$f^*(x)$ 

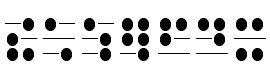
If such a sign is printed in the subscript position, the subscript sign should be used unless this degree of explicitness is not required (in which case it may be coded without as above). When the subscript sign is used, the index terminator should be inserted before following unspaced expressions not in the same subscript position, as usual. (See also the note in §**2.2.5** concerning subscript bars and tildes used as fount signs.)

Ex.17 $f_*(x)$ 

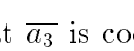
$\underline{x}$ 

$\underline{xy}$ 

$\underline{x}_n$ 

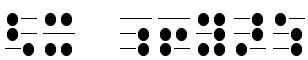
$\underline{\int} f dx$ 

6.9.1 In geometry, the print notation $\widehat{ABC}$ or $A\hat{B}C$ (the angle ABC) is exceptionally more conveniently coded as $\angle ABC$, rather than using the hat sign.

6.9.2 Bars, hats, etc. in braille are to be understood as referring only to the immediately preceding symbol or bracketed group; brackets should thus be inserted in braille to show that such a bar, hat, etc. extends over several signs in print. (As in §**6.8**, lower numbers are not regarded as full signs here but as being part of the sign to which they are an index, so that $\overline{a_3}$ is coded  and $\overline{a^2}$ as , etc. Special functions (see §**7**) are also regarded as single entities in this connection.)

Ex.18 $x\bar{y}$

$\overline{AB}$

$\overline{x+y}$ 

$\overline{x_n}$

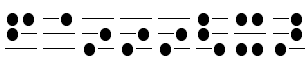
$a_{\bar{3}}$

$x_{\bar{n}}$

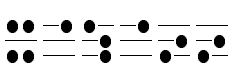
$\overline{\frac{3}{4}}$

6.9.3 An uninterrupted string of dashes or stars is coded by giving only the initial dot 4.

Ex.19 x''

$f'''(x)$ 

y^{**}

$\hat{x}''$ 

For other such combinations each dash, star, etc. should have its own dot 4 or dots 45.

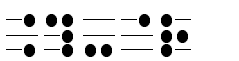
Ex.20 x'^{*}

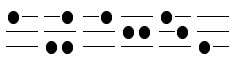
$\hat{\hat{x}}$

$\hat{x}'$

6.10 The signs , , , etc. are used to indicate respectively single, double, triple dots, etc. above a letter or symbol. A terminator is not required after such signs.

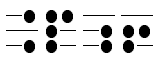
Ex.21 $\dot{x}$

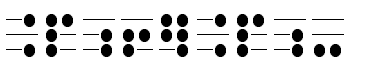
$\ddot{\theta}\mathbf{r}$ 

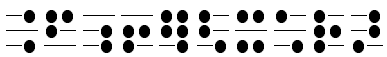
$\dot{\overleftarrow{a}}$ 

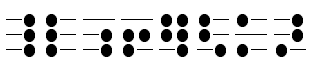
6.11 A + or − sign in an index position may be coded without the superscript or subscript sign by placing the + or − close up to the term. The sign should be used to separate such a term from an immediately following expression.

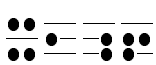
Ex.22 e^-

π^+ 

$\pi^+\pi^-$ 

$\phi^+(x_r)$ 

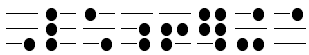
$\Lambda_+(k)$ 

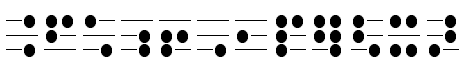
x_1^+ 

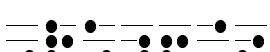
M_+^2

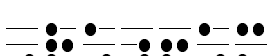
R_+^*

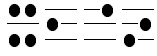
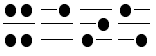
The superscript or subscript sign should, however, be used for such expressions if ambiguities arising from non-index + or − signs could occur (e.g. to distinguish from the expression $x \to 0+$), or if greater explicitness is required.

Ex.23 $L_+^{\uparrow}$ 

$\phi_{+,p}(x)$ 

$R_{+'}$ 

R_{+_m} 

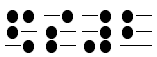
6.12 When several indices are attached to a term they should be brailled in the most natural order according to their meaning, i.e. as they would normally be read. In particular, for the $\int$ integral sign, $\sum$ summation sign, $\prod$ product sign, $\bigcup$ union sign, $\bigcap$ intersection sign, etc., subscripts should be brailled before superscripts. (See the various examples in §**6.2**.) When such considerations do not clearly apply, indices should then simply be stated in the most convenient order for coding. In particular, it is generally neatest to braille subscript numbers attached to letters before dashes, bars, etc., as the subscript sign can thereby be dispensed with, e.g. braille x_1' as  rather than  .

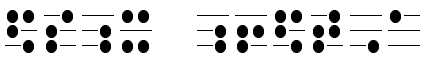
The order of indices should not otherwise be regarded as critical, though once a conventional order for a type of expression has been established in a particular piece of work, it should be adhered to throughout for consistency.

7 SPECIAL FUNCTIONS AND OTHER WORDS AND ABBREVIATIONS

7.1 These are words, parts of words, or sets of initials used to denote functions, etc., and are generally printed so as to be distinguishable from algebraic letters (i.e. usually printed in ordinary type rather than italic type for algebraic letters). Such functions, etc. are indicated in braille by placing the sign before the letter or letters which denote it. Some of the more common functions (e.g. sin, cos, log) have special braille abbreviations as given in the table of signs; in other cases the word segment is brailled in full without literary contractions.

A prefixed function or expression may be immediately followed by any mathematical sign; an immediately following small Latin letter thus requiring its dots 56 letter sign. (This latter rule applying also to the signs for $\triangle$ and for $\square$.)

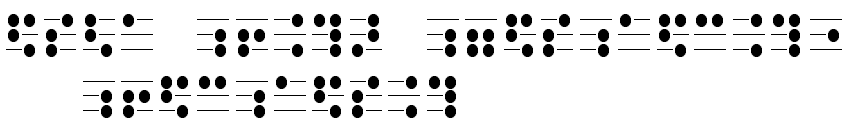
Ex.1 $\sin 2$ 

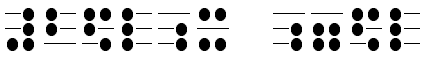
$\sin x + \tan A$ 

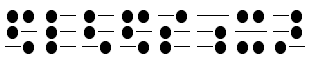
$\cosh x$

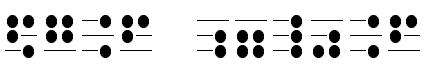
$\operatorname{sech} x$

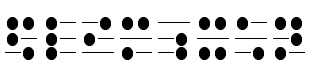
$\sin(a + \theta) = \sin a \cos\theta + \cos a \sin\theta$

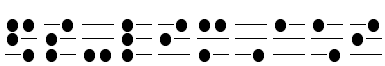


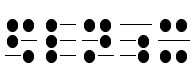
$2 \log x = \log x^2$ 

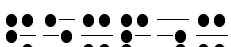
$\log(\sin x)$ 

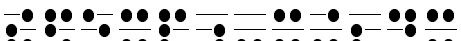
$\operatorname{grad} \phi = \nabla \phi$ 

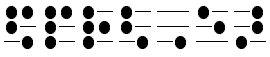
$\lim x_n$ 

s-lim A_i 

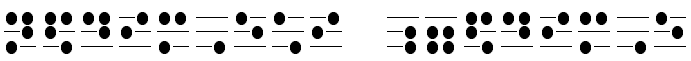
$\ln x$ 

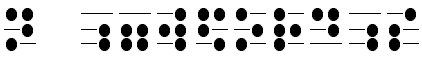
$\exp x$ 

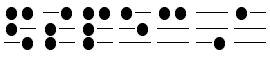
$\int \exp -x^2 dx$ 

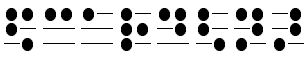
$\Pr(E)$ 

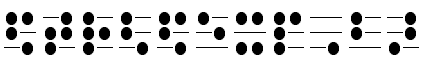
$z = \operatorname{Re} z + i \operatorname{Im} z$

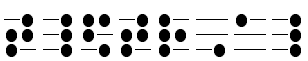
$n \dim E_i = \dim E$ 

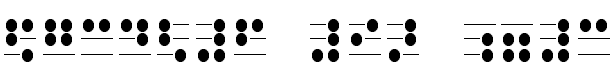
$n = w \operatorname{ord} s$ 

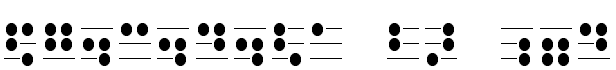
$\operatorname{Spec} A$ 

$\operatorname{Card}(n)$ 

$\operatorname{tr}(\exp B)$ 

$t|\operatorname{tr} A|$ 

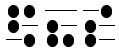
$\gcd(6, 9) = 3$ 

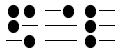
g.c.d.$(a, b) = d$ 

7.1.1 A function with a special braille abbreviation may be used to represent a variety of print forms with the given meaning.

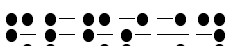
Thus

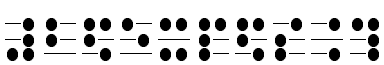
represents $\sin^{-1}$, arcsin, arc sin;

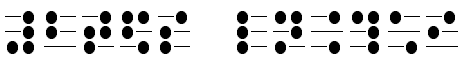
 ... $\sinh^{-1}$, arcsinh, arsinh;

 ... $\log^{-1}$, antilog, etc.

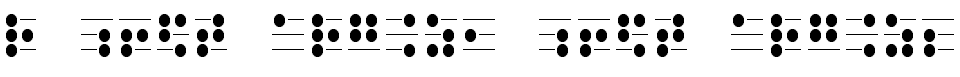
7.2 A sequence of two or more spaced word segments (which may denote separate functions, or together a single function), not interrupted by any other mathematical sign, is generally coded with a single initial sign and with the spaces between word segments reproduced. A function within the sequence which has a special braille abbreviation should, however, be prefixed by its own sign, with the space separating it from the preceding word segment omitted.

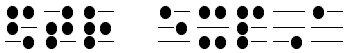
Ex.2 $\log \sin \theta$ 

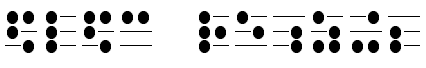
$2 \exp \sin \theta$ 

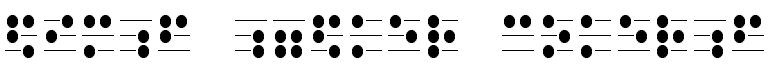
$2t \sin \ln \theta_i$ 

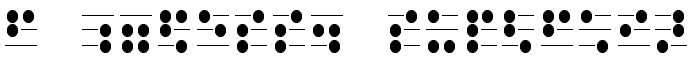
$r + \tan \arg \zeta_1 + \tan \arg \zeta_2$



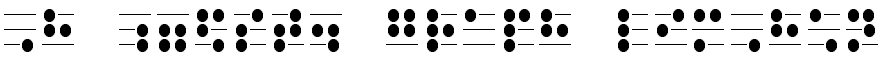
$\operatorname{tr} \exp A$ 

$\log \cos \operatorname{Re} z^2$ 

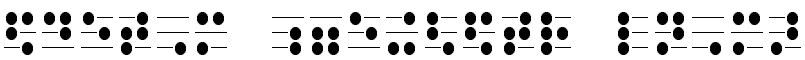
$\operatorname{im} f = \ker \operatorname{coker} f$ 

$f = \text{ess. sup}(f_i)$ 

$H = \text{st. graph} \lim H_n$

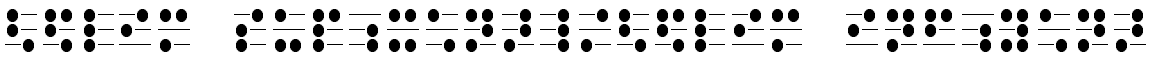


$\det M = e^{\operatorname{tr} \ln M}$



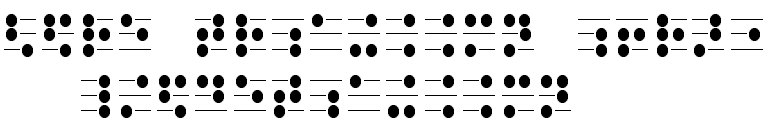
(brackets are required according to §**6.5**)

$$\frac{\limsup x_n}{\liminf y_n}$$

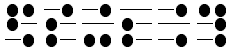


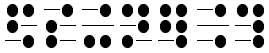
(brackets are required according to §**4.1** (a))

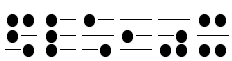
$$\frac{\operatorname{Re} \operatorname{tr} a^{\mu\nu} + r}{\det a^{\mu\nu}}$$

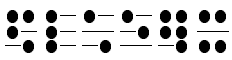


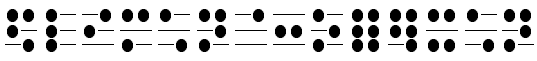
7.3 Indices are generally coded in the usual way with the superscript or subscript signs (the subscript sign should not be omitted before lower numbers here, to avoid them being misread as punctuation marks). The inverse logarithmic, trigonometric and hyperbolic functions, however, have special designated braille signs (e.g. represents $\sin^{-1}$, etc.).

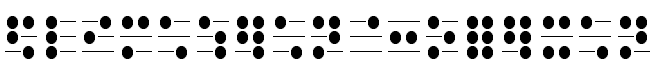
Ex.3 $\sin^2 \theta$ 

$\sin^n \theta$ 

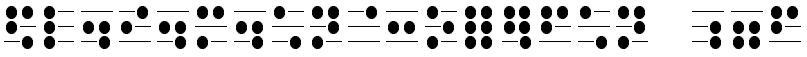
$\log_{10} x$ 

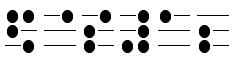
$\log_e x$ 

$\lim_{n\to\infty} x_n$ 

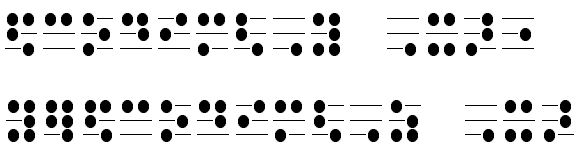
$\varliminf_{n\to\infty} x_n$ 

l.i.m.$_{n\to\infty} f_n = f$

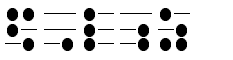


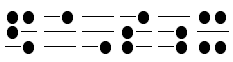
$\sin^{-1} \frac{1}{2}$ 

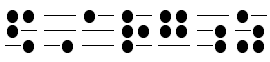
$\cosh^{-1} x$

$\operatorname{codim}(Y, X) = \inf_{Z\subseteq Y} \operatorname{codim}(Z, X)$ 

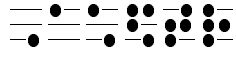
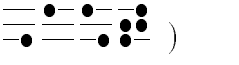
7.4 Functions beginning with an initial capital in print may be brailled by inserting a dot 6 before the initial letter after the sign, when this use of capitals in print is mathematically significant.

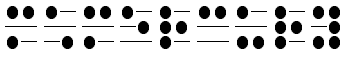
Ex.4 $\operatorname{Log} z$ 

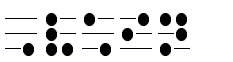
$\operatorname{Sin}^{-1} x$ 

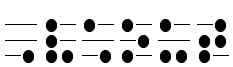
$\operatorname{Arg} z$ 

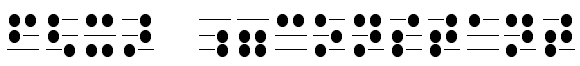
7.5 The sign may be used generally to indicate and distinguish ordinary words or abbreviations within mathematical expressions, though a certain degree of discretion should be exercised here, since in many cases (particularly with indices) the expression is more simply coded and read without the sign by just leaving the expression uncontracted.

Ex.5 A_{tr}  (as distinct from A_t )

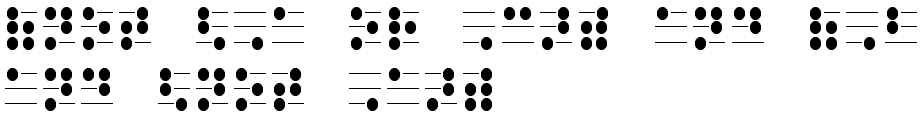
m_{mercury} 

V_{in} 

V_{out} 

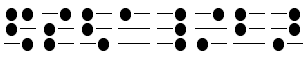
$f(x) = \text{constant}$ 

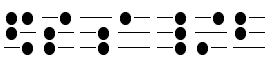
[not (A or C)] and [B and (not A)]

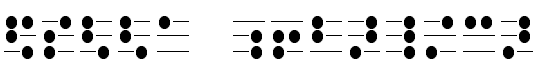


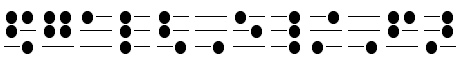
8 DELIMITING THE ARGUMENT OF A FUNCTION

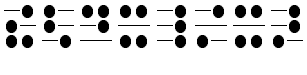
8.1 If the argument of a function, integral sign, summation sign, etc. is a fraction brailled with a sign, the argument must be enclosed in brackets in braille whether or not this is done so in the print. Brackets should not be inserted where the print is ambiguous in such cases.

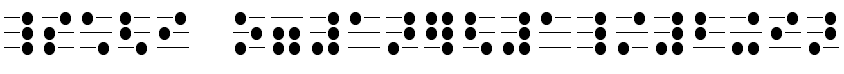
Ex.1 $\sin \dfrac{a}{b}$ 

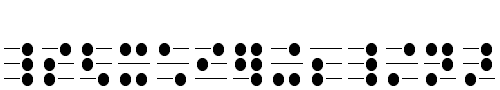
$\dfrac{\sin a}{b}$ 

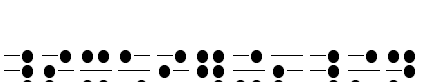
$\sin \dfrac{a+b}{c}$ 

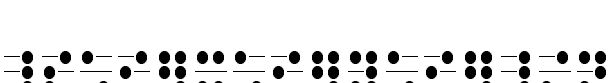
$\text{Gal}\frac{E}{F}$ 

$\displaystyle\int \frac{dx}{x}$ 

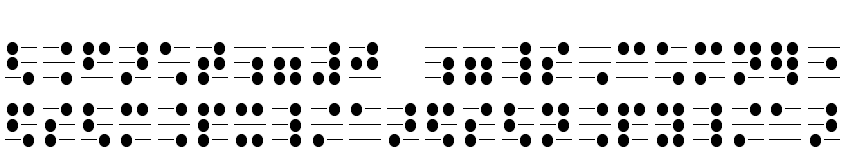
$\displaystyle\sum_{i\geq 1} \frac{1}{2^i}$ 

$\displaystyle\sum \frac{x_i^2}{n}$ 

$\dfrac{\sum x_i^2}{n}$ 

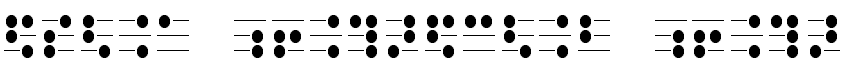
$\displaystyle\sum_i x_i y_i / n$ 

$(\phi)_{t=0} = \sum C_{mn} \sin m\pi x/a \ \sin n\pi y/a$

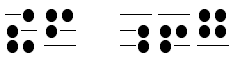


8.1.1 If spacing is used in print to indicate that the argument consists of two or more terms which are spaced in braille, that indication of extent should be shown in braille by enclosing the whole argument in brackets. Brackets should not be inserted where the print is ambiguous in such cases.

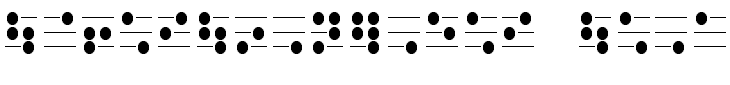
Ex.2 $\sin\alpha+\theta\ \cos\beta+\theta$



$\int f+g$



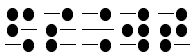
$\bigcup_{i\in N} E_i \setminus A$



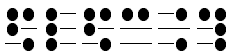
(Cases such as log cos Re z^2 are brailled without inserting brackets in accordance with §**7.2**.)

8.1.2 It is not necessary to insert additional brackets for arguments consisting of unspaced terms in braille. The conventional print use or non-use of brackets should be sufficient in such cases.

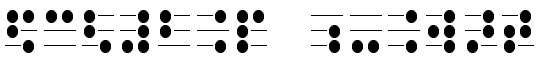
Ex.3 $\sin\omega t$



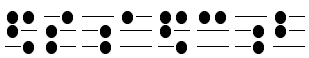
$\log\cos\theta$



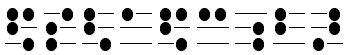
$\cos(2\pi-\omega)t$



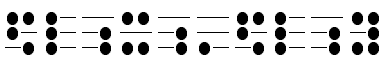
$\sin a\cos b$



$\sin(a\cos b)$

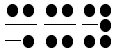


$\log x\cdot\log y$

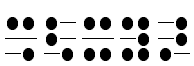


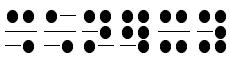
8.2 The root sign is generally to be understood as applying only to the immediately following number, letter, function or bracketed group (i.e. that following the root exponent if present). Thus if a large root sign is used in print to show that it applies to a fraction which would be brailled with the sign, or if a horizontal bar is used in print to show that the argument extends beyond the first letter or number, brackets should be placed around the whole argument in braille if they are not already present in the print. Indices may, however, be attached to the argument without the use of brackets when the meaning is clear (in particular, when brackets or the horizontal bar is not used in print).

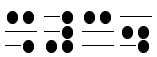
Ex.4 $\sqrt{xy}$

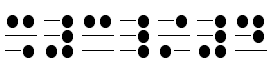


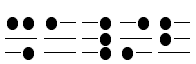
$\sqrt{xy}$

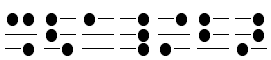


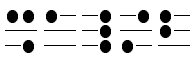
$\sqrt[n]{xy}$ 

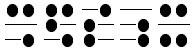
$\sqrt{\frac{3}{4}}$ 

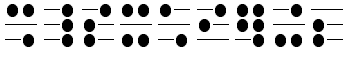
$\sqrt{3/4}$ 

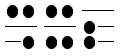
$\frac{\sqrt{a}}{b}$ 

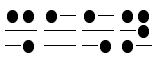
$\sqrt{\frac{a}{b}}$ 

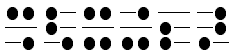
$\sqrt{a/b}$ 

$\sqrt{\sin x}$ 

$\sqrt{\sum x_i^2}$ 

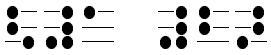
$\sqrt{x_2}$ or $\sqrt{x_2}$ 

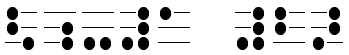
$\sqrt{a_n}$ 

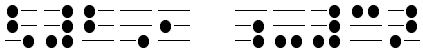
$\sqrt{x^2}$ 

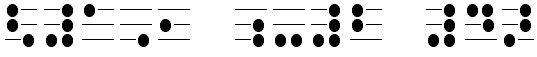
9 COORDINATES AND SETS

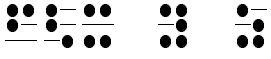
9.1 Commas shown in print to separate elements (or arguments) of sets, coordinates, functions, etc. displayed in brackets, are generally omitted in braille, the spacing between those elements being sufficient. A comma before an element preceded by an operation sign (usually + or −) should, however, be retained to avoid ambiguity.

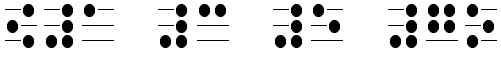
Ex.1 $(1,2)$ 

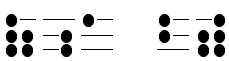
$(-1,2)$ 

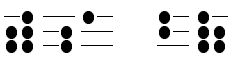
$(2,-3)$ 

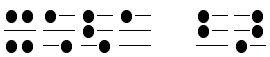
$(1,-2,4)$ 

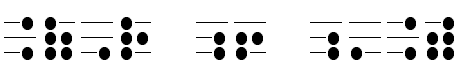
$f(x,y,z)$ 

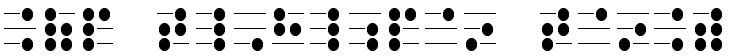
$\{1,3,5,7\}$ 

$[a, b]$ 

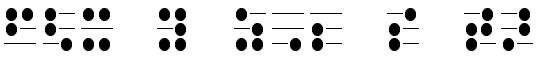
$]a, b[$ 

$x_{(a,b)}$ 

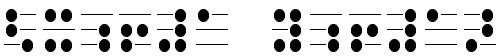
$< R, +, \cdot >$ 

$< p, t|H|p', t' >$ 

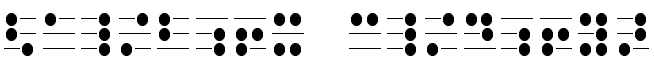
9.1.1 When the arguments of a function generally printed with commas are separated into groups by a different punctuation mark (e.g. a semicolon), this latter punctuation mark should be retained in the braille.

Ex.2 $f(x, y, z; s, t)$ 

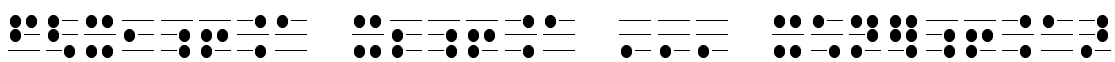
9.1.2 Elements consisting of two or more terms which would normally be spaced in braille, may be united in such expressions by omitting the spaces between terms (if not already united by being enclosed in brackets). In omitting spaces it will be necessary in some cases to insert a sign to terminate an index.

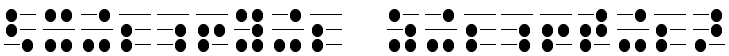
Ex.3 $(x + 1, y + 1)$ 

$(x + t, -y, z)$

$(\frac{a}{b} + x, \frac{c}{d} + y)$ 

$f(x_1 + \alpha, x_2 + \alpha, \ldots, x_n + \alpha)$

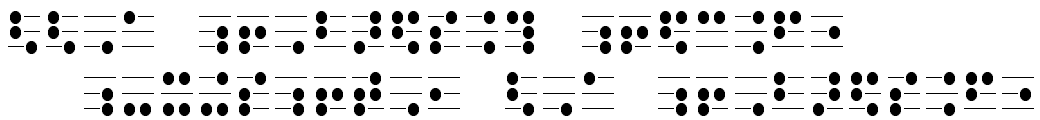


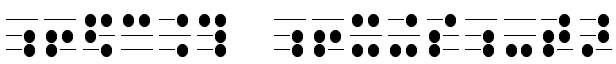
$(x^2 + y^2, z^2 + t^2)$ 

$((x + \alpha), (x - \alpha))$

In particularly complex cases it may be preferable to preserve the spaces between terms by retaining the commas between elements. This method may also be necessary when an element contains spaced word segments (§**7.2**), since these spaces may not be omitted.

Ex.4 $((A + B)\sin\theta + \cos\phi - x^{s+t}, (A + B)\sin\phi + \cos\theta + x^{s-t})$

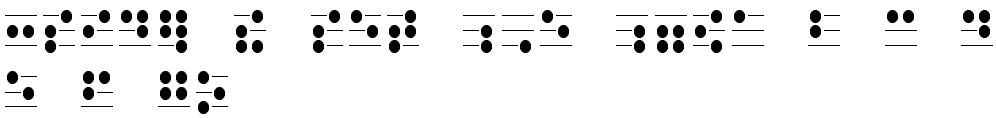




9.1.3 An element with spaces omitted to unite it according to §**9.1.2** should not be split between terms at the end of a braille line: the element should rather either be brailled with spaces by enclosing it in brackets (so that it can be split between terms), or else it should be taken down complete onto the next line.

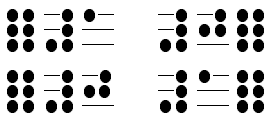
9.2 When sets or coordinates are divided between elements at the end of a braille line, a dot 5 mathematical hyphen should not be used (as it would imply that the element continued), nor should a comma be used if commas are generally omitted in the braille.

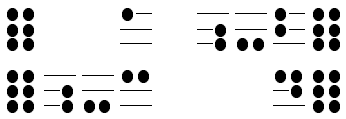
Ex.5 Consider the set $E = \{a, b, c, d, e, f, g\}$

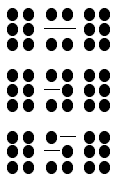


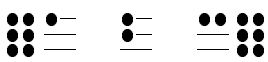
10 MATRICES, DETERMINANTS AND OTHER ARRAYS

10.1 Round or square matrix brackets are brailled as columns of signs of the appropriate length. Long vertical bars (e.g. for determinants) or double vertical bars are brailled as single or double columns of signs respectively. Columns of elements are spaced from each other by 1 clear cell running down between the columns. Elements within a column should be brailled with their left-hand cells aligned unless prefixed by a + or − sign which is normally brailled to stand out. Elements beginning with a small Latin letter do not need to be prefixed by a letter sign unless they immediately follow a vertical bar sign.

Ex.1 $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ 

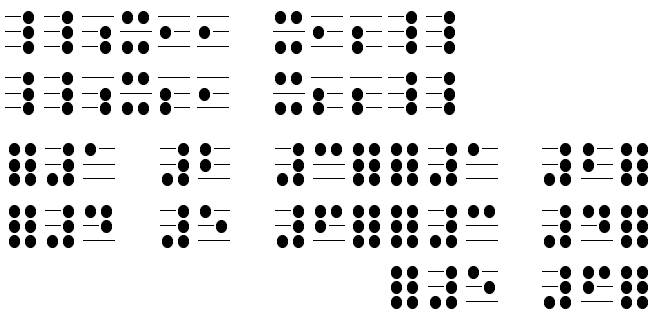
$\begin{bmatrix} a & -b \\ -c & d \end{bmatrix}$ 

$\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ 

$(\,a \quad b \quad c\,)$ 

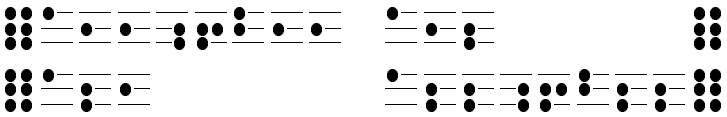
$$\left\| \begin{matrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{matrix} \right\|$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$$



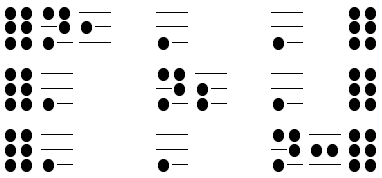
10.2 Elements consisting of two or more terms which would normally be spaced in braille are united by omitting the spaces as is done with coordinates or sets (§**9.1.2**).

Ex.2 $$\begin{bmatrix} a_{11}+b_{11} & a_{12} \\ a_{21} & a_{22}+b_{22} \end{bmatrix}$$

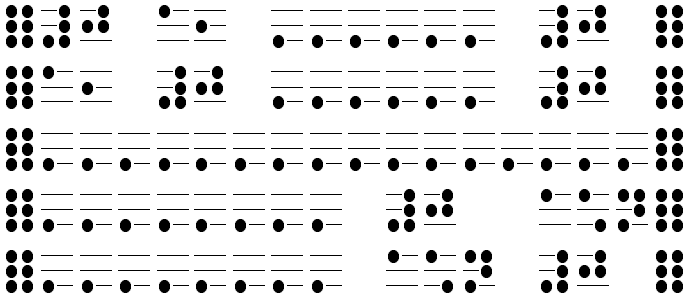


10.3 A site in a matrix marked with a dot in print should be marked with a dot 3 in braille. Ellipses are brailled as ordinary literary ellipses which should be aligned in an appropriate matrix column, and a line of dots is brailled as a line of dot 3's.

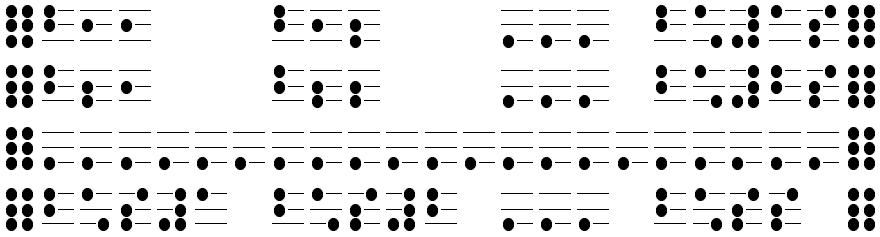
Ex.3 $$\begin{bmatrix} n_1 & \cdot & \cdot \\ \cdot & n_2 & \cdot \\ \cdot & \cdot & n_3 \end{bmatrix}$$



$$\begin{bmatrix} 0 & a_1 & \ldots\ldots\ldots & 0 \\ a_1 & 0 & \ldots\ldots\ldots & 0 \\ \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots \\ \ldots\ldots & & 0 & a_n \\ \ldots\ldots & & a_n & 0 \end{bmatrix}$$



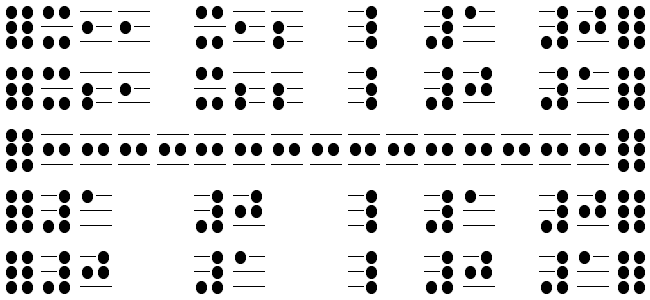
$$\begin{bmatrix} b_{11} & b_{12} & \ldots & b_{1s} \\ b_{21} & b_{22} & \ldots & b_{2s} \\ \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots \\ b_{s1} & b_{s2} & \ldots & b_{ss} \end{bmatrix}$$



10.4 Partition lines in matrices may be shown by a sequence of signs for horizontal lines, and by signs aligned vertically for vertical lines.

Ex.4

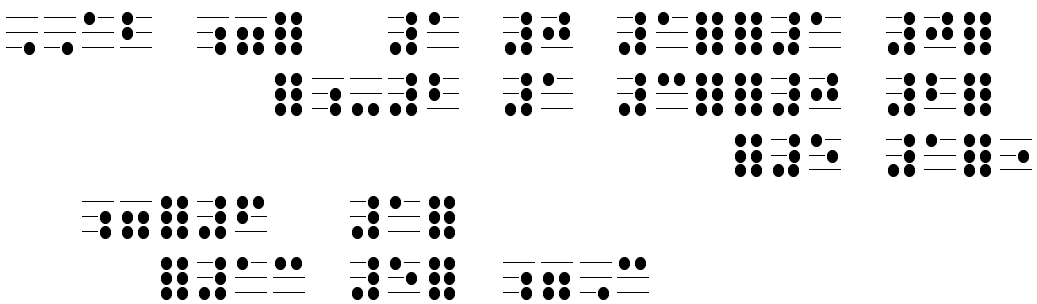
$$\left(\begin{array}{cc|cc} x_{11} & x_{12} & 1 & 0 \\ x_{21} & x_{22} & 0 & 1 \\ \hline 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{array}\right)$$



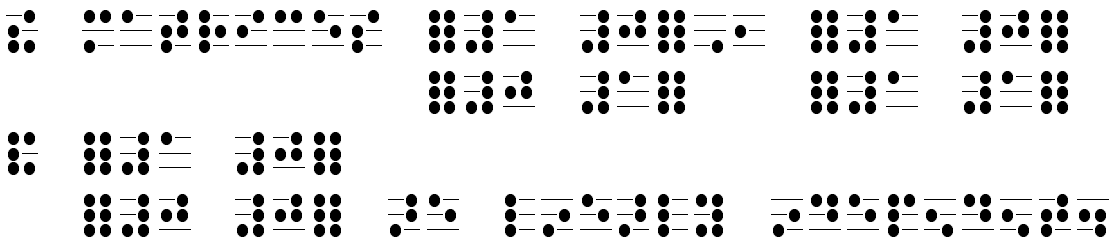
10.5 Matrices or determinants set out on a separate line in print are normally indented to begin in cell 5 (or cell 7 for runovers to an equation) as usual for mathematics. A cell 1 start may be used for wide matrices if it allows the array to be brailled without runovers when this would not be possible with the normal indentation.

In a matrix equation or for matrices within ordinary text, all the matrices which fit across the page should be brailled with their top rows and any other intervening single line expressions or text on the same braille line. Single line expressions or text following a matrix should be brailled on the bottom line of the matrix unless another matrix appears on that same line in which case the expression or text is placed on the top line between the matrices.

Ex.5 $AB = \begin{bmatrix} 1 & 0 & 1 \\ -2 & 1 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 2 \\ 5 & 1 \end{bmatrix} = \begin{bmatrix} 6 & 1 \\ 13 & 5 \end{bmatrix} = C$



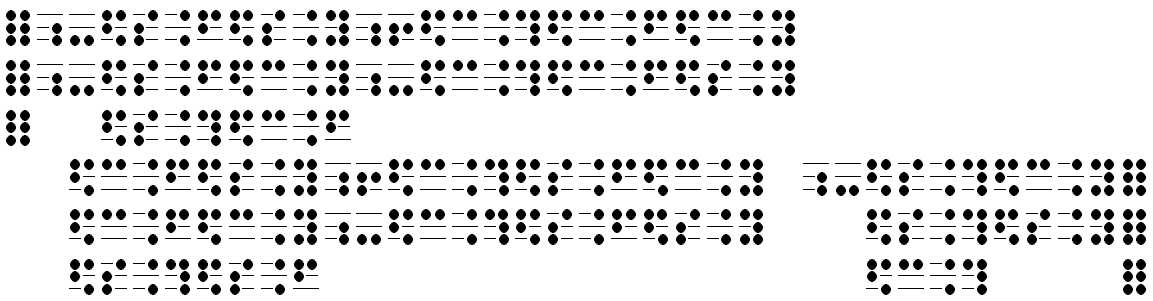
Ex.6 The matrices $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$ and $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ are linearly independent.



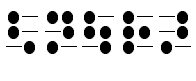
Matrices should not be brailled directly beneath one another with one or both of the long brackets aligned. To avoid this one of the matrices can be moved on 1 or 2 cells, or else a blank line left between the matrices.

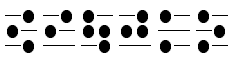
10.6 A matrix which is too wide for the braille page may be split between columns with the remaining part of the matrix placed beneath and indented 2 cells from the start of the matrix. Facing pages may also be used for wide matrices if convenient.

Ex.7
$$\begin{bmatrix} -\sin\phi\sin\psi + \cos\theta\cos\phi\cos\psi & \cos\phi\sin\psi + \cos\theta\sin\phi\cos\psi & -\sin\theta\cos\psi \\ -\sin\phi\cos\psi - \cos\theta\cos\phi\sin\psi & \cos\phi\cos\psi - \cos\theta\sin\phi\sin\psi & \sin\theta\sin\psi \\ \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \end{bmatrix}$$

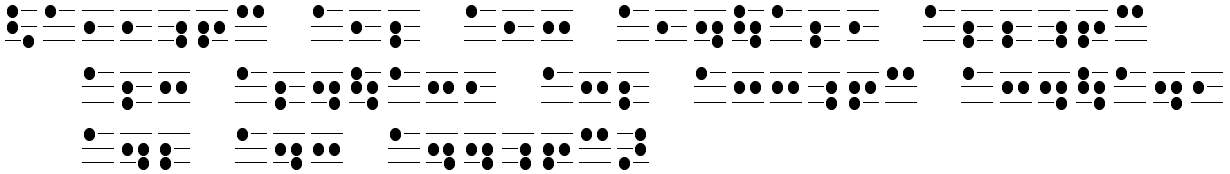


10.7 A linear method may alternatively be used to represent matrices and other such arrays. This method is favoured for binomial coefficients, and for Christoffel symbols whereby the particular type of bracket used in print can be represented in braille, but it is not the primary method used for representing matrices in general transcription work. In this method the rows are brailled in order, from the topmost downwards, with an unspaced sign used to indicate the start of a new row. The whole sequence is enclosed in the appropriate standard mathematical brackets, and may be divided at any convenient point at the end of a braille line, a dot 5 mathematical hyphen not being used if the division is between spaced elements.

Ex.8 $\binom{n}{r}$ (a binomial coefficient) 

Ex.9 $\left\{ \begin{matrix} i \\ jk \end{matrix} \right\}$ (a Christoffel symbol) 

Ex.10
$$\begin{pmatrix} a_{11}+c & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22}+c & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33}+c & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44}+c \end{pmatrix}$$

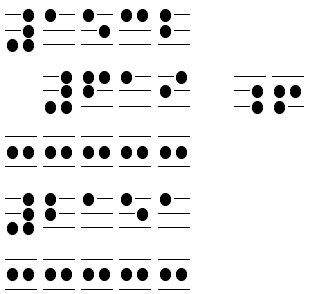


11 WORKED CALCULATIONS; LOGICAL FIGURES

11.1 Horizontal lines used in worked calculations are brailled as a line of signs of the appropriate length. Digits, tens and hundreds, etc. should generally be vertically aligned, and operation signs placed as in print. It may be convenient to omit numeral signs so as to leave the braille less cluttered, but this convention should be explained beforehand in general transcription work.

Ex.1

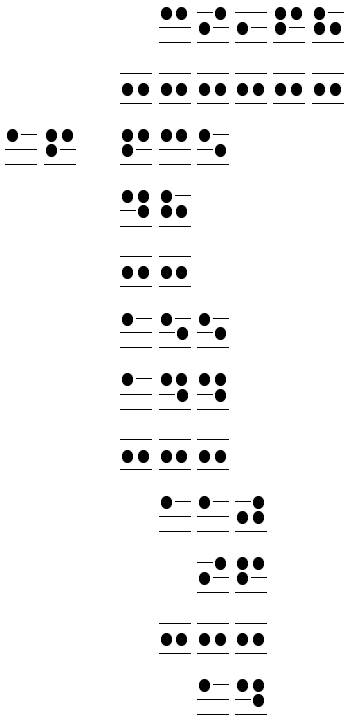
```
 1532
  619 +
 ----
 2151
 ----
```



Ex.2 [Numeral signs are omitted]

```
      39.68
16 |635
    48
    ---
    155
    144
    ---
     110
      96
     ---
      14
```

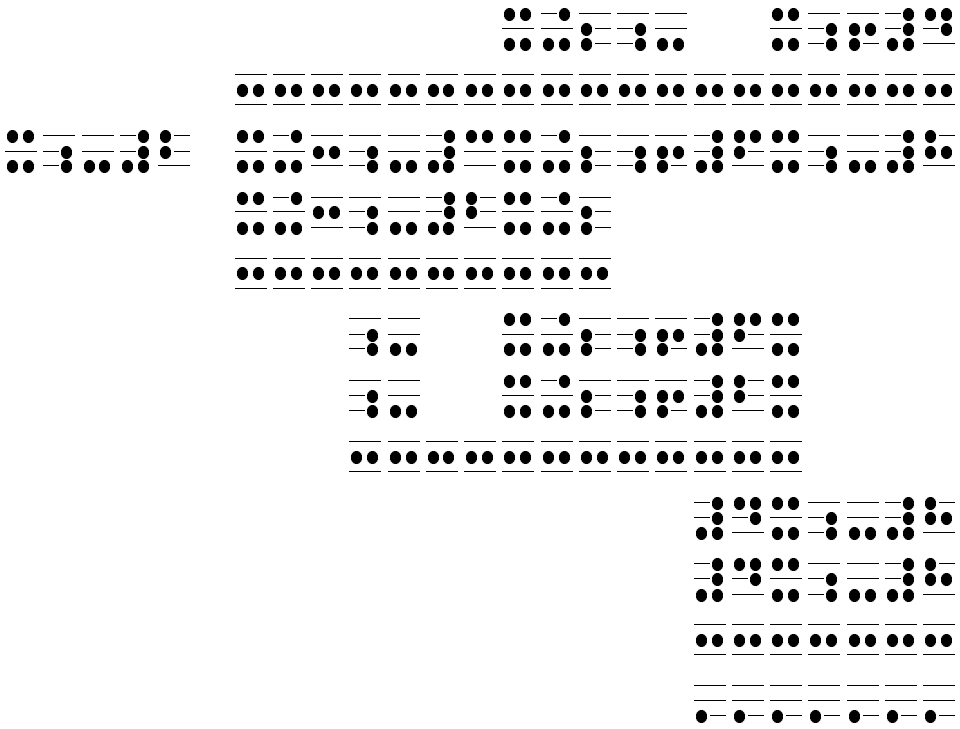
etc.



11.2 Worked calculations with algebraic expressions should generally be arranged so that the + and − signs and terms of the same degree are vertically aligned. This may be achieved compactly by omitting spaces (as well as possibly leaving or inserting spaces where necessary), or less compactly by just inserting the necessary spaces.

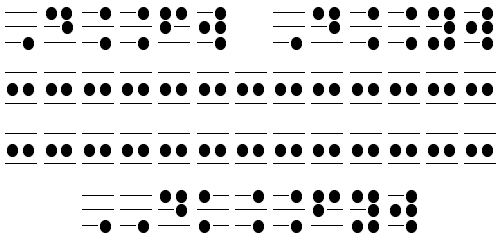
Ex.3

$$
\begin{array}{r|l}
 & x^2 - x + 4 \\
\hline
x - 2 & x^3 - 3x^2 + 6x - 8 \\
 & x^3 - 2x^2 \\
\hline
 & -x^2 + 6x \\
 & -x^2 + 2x \\
\hline
 & 4x - 8 \\
 & 4x - 8 \\
\hline
\end{array}
$$



11.3 Horizontal lines in logical figures may also be represented by a sequence of signs.

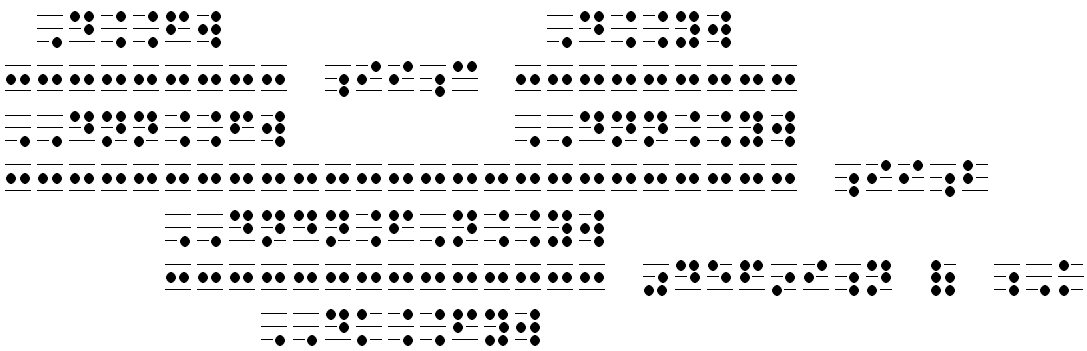
Ex.4 $\dfrac{D\phi\omega \qquad D\psi\omega}{DK\phi\psi\omega}$



Ex.5

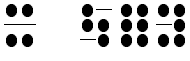
$$\dfrac{\dfrac{D\phi\omega}{DNN\phi\omega}\ \text{IIc}\ \dfrac{D\psi\omega}{DNN\psi\omega}}{\dfrac{DNDN\phi N\psi\omega}{DK\phi\psi\omega}}\ \text{IIb}$$

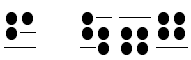
by definition of K



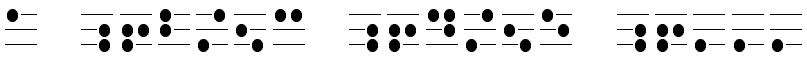
12 MISCELLANEOUS NOTATION

12.1 The signs and are used to represent a variety of operator symbols as required.

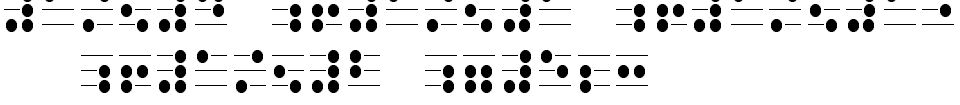
Ex.1 $x * y$ 

$f \circ g$  (function composition)

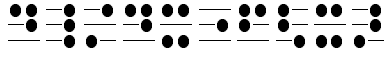
12.2 Continued fractions are coded using as a modified fraction line sign indicating that all the terms following it are part of the denominator. (This notation should not be used in other contexts.)

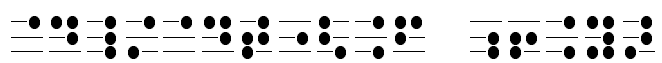
Ex.2 $a+\cfrac{b}{c+\cfrac{d}{e+\ldots}}$ 

Ex.3 $\cfrac{1}{4+}\,\cfrac{1}{1+}\,\cfrac{1}{1+}\,\cfrac{1}{2} = \cfrac{5}{23}$

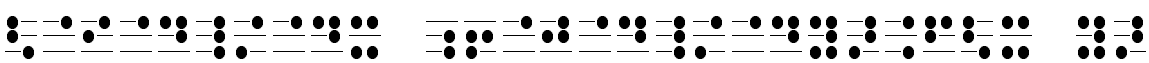


12.3 The differential operators $\dfrac{d}{dx}$, $\dfrac{\partial}{\partial x}$, $\dfrac{\partial^2}{\partial x \partial y}$, etc. should be separated from their following operands by a dot 5 if they (the operators) are not enclosed in brackets.

Ex.4 $\dfrac{d}{dx} f(x)$ 

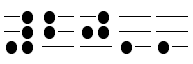
$\frac{\partial}{\partial t}(\phi + \psi)$ 

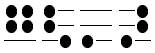
$(\mathbf{i}\frac{\partial}{\partial x} + \mathbf{j}\frac{\partial}{\partial y})\phi(x, y)$

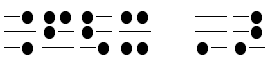


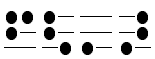
($\frac{df}{dx}$, $\frac{\partial f}{\partial x}$, etc. are coded as ordinary fractions.)

12.4 A dot or blank in print indicating a missing digit or an unspecified argument, etc, may be shown in braille as a single dot 3.

Ex.5 20.. 

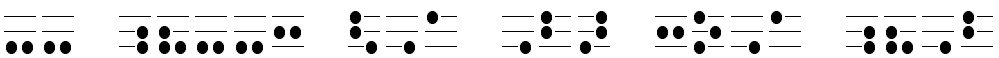
$g(.)$ 

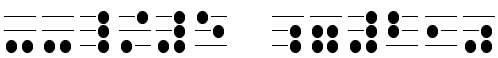
$\phi(x, .)$ 

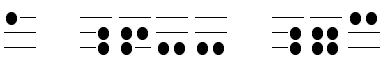
$f(\ )$ 

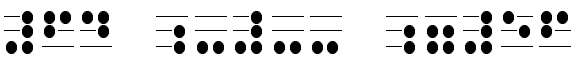
12.4.1 Dashes are also freely used for similar purposes. However, it is usually inadvisable for the dash to immediately follow a minus sign; if in this case the dash represents a number, a numeral sign can be inserted before it.

Ex.6 $— \times — :\ (A, B) \to A \times B$

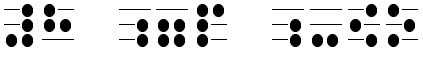


$\overline{5} = \frac{2}{10}$ 

$a + ? = c$ 

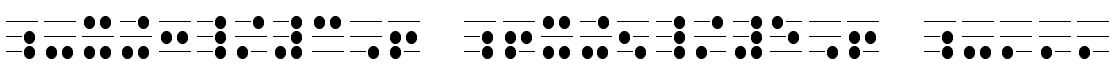
$64 - ? = 56$ 

A general alternative in cases like the last two examples is to substitute a symbol which is unambiguous in the context, for example a literary dagger. An explanation of the method would then be needed for the reader.

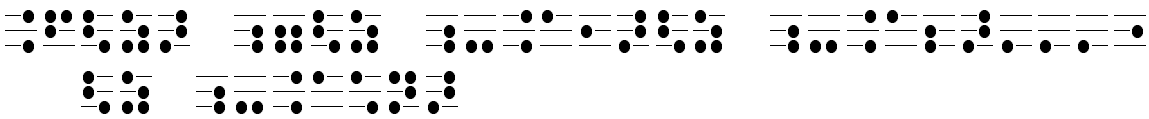
Ex.7 $8 = p - ?$ 

12.5 Ellipses representing omitted or unspecified terms in an expression, are spaced in the same way as the terms that they represent would be. (An ellipsis is coded as an unspaced sequence of three dot 3's.)

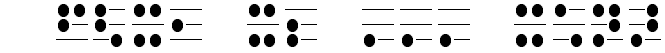
Ex.8 $\sin x = x - x^3/3! + x^5/5! - \ldots$



$\phi(z) = (z - \alpha_1)(z - \alpha_2)\ldots(z - \alpha_n)$

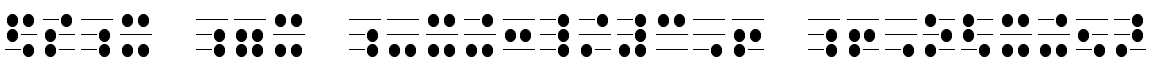


$f(x_1, x_2, \ldots, x_n)$

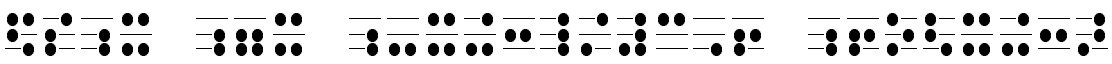


12.6 The (Landau) notation $O(y)$ and $o(y)$ used in analysis, etc, should be coded with letter *o*'s (i.e. not zeros).

Ex.9 $\sin x = x - x^3/3! + O(x^5)$



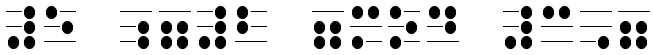
$\sin x = x - x^3/3! + o(x^3)$



(for small x)

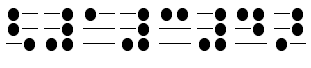
12.7 In expressions such as $5 = 2 \pmod 3$, the bracketed remark should be spaced from the preceding term, and literary brackets used.

Ex.10 $5 = 2 \pmod 3$

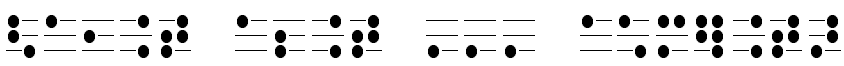


12.8 In the notation (below) for cyclic permutations, the numbers or other symbols used may be brailled spaced or unspaced according to print. Each number should have its own numeral sign, whether spaced or unspaced.

Ex.11 (134)

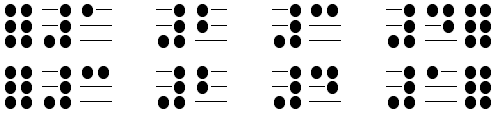


$(a_1\tau\ a_2\tau\ \ldots\ a_m\tau)$



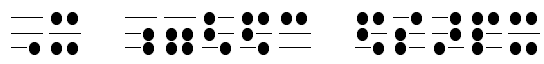
The notation for a general permutation may be brailled as a matrix.

Ex.12 $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 2 & 4 & 1 \end{pmatrix}$



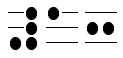
12.9 The notation $\begin{matrix}\cos\\ \sin\end{matrix} x$ (meaning $\cos x$ or $\sin x$) etc. may be coded on one line by using brackets.

Ex.13 $X = \overset{\cos}{\underset{\sin}{\cdot}} px$

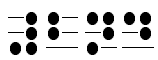


12.10 The terminal letters in expressions such as $\frac{1}{3}$rd and $\frac{3}{5}$th's should generally be omitted in braille after fractions (but not after whole numbers).

Ex.14 $\frac{1}{3}$rd



2nd



13 LAYOUT OF MATHEMATICAL TEXT

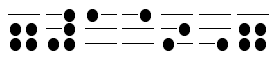
The following remarks are additional to those in §**1**.

13.1 Centred headings should not begin before cell 9 to avoid confusion with set out mathematics.

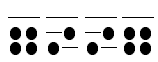
13.2 Equation numbers should be placed in literary brackets (whether or not brackets are used in the print), starting in cell 5 of the first line of the equation or equations to which they refer. If any further line of an equation has a reference number, it should be placed (in brackets) at the beginning of the line to which it refers, starting in cell 5, and followed by two blank spaces before the equation continues.

If the reference symbol is not a simple number but wholly or partly some other sign, it is treated in the same way, with a dot 6 separation sign inserted before the closing bracket if necessary.

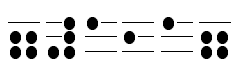
Ex.1 $(1')$



$(*)$



(1.1)



13.3 The following are cases where it may be necessary or advantageous to differ from the usual cell 5 with runovers in cell 7 mathematics layout:

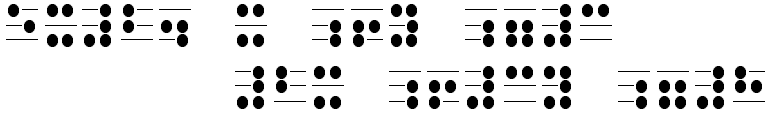
13.3.1 Wide arrays (§**10**).

13.3.2 Worked calculations (§**11**).

13.3.3 To enable simultaneous equations to all start in the same cell if the first equation is preceded by an equation number or otherwise does not begin in cell 5.

Ex.2 $x + y = 3$

$2x + 3y = 8$

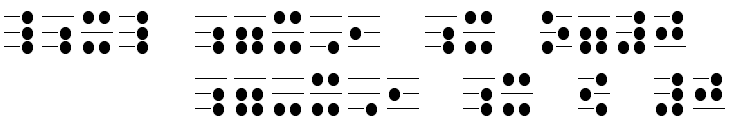


13.3.4 To allow cell 5 or cell 7 indented text indicators (e.g. question numbers or letters) to stand out in parts of text consisting principally of such indicators and set out equations:
(a) The layout used in §**13.3.3** above may be used where applicable; or
(b) The equations may each be started in cell 1; or
(c) Blank lines may be inserted to separate the numbered or lettered sections of text, retaining the usual cell 5 and 7 layout of equations and those numbers or letters.

Note: Set out equations not beginning in cell 5 by virtue of §**13.3.3** or §**13.3.4** above should not be allowed runovers. If equations are thereby too long (i.e. any equation of a set of simultaneous equations), a standard cell 5 and 7 format should be adhered to instead. (Equations simply preceded by cell 5 equation numbers are normally allowed cell 7 runovers.)

13.4 If the right-hand side of an equation has several options subject to different conditions (often grouped with a large bracket in print), the large bracket is omitted in braille, and one of the following alternative layouts may be used:
(a) The left-hand side and first option may be brailled as a complete equation, and the second and further options placed beneath with their equals signs in the same cell as that of the first option, if the options are all short enough to fit without runovers; or
(b) The left-hand side of the equation (if it is reasonably short) may be repeated for each option, so that the options are each brailled as complete equations; or
(c) The left-hand side may be brailled once followed by the first option, with the subsequent options each begun on a separate line with their equals signs in cell 5, and with all runovers in cell 7.

Ex.3 $$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$



13.5 If conditions or words of comment are associated with and interrupt a mathematical expression, they may be inserted as in print at the appropriate place, but should not be preceded or followed by a dot 5 hyphen. The mathematical expression should continue in cell 7 of a new line.

14 UNITS

(Tables of standard unit abbreviations are given at the end of this section for reference.)

(*Note*: The examples in this section have been chosen to illustrate a variety of print forms as found: this choice is not intended to indicate recommended practice.)

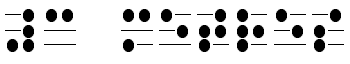
14.1 Units are placed before or after the number to which they refer, according to print. Units should be spaced in braille, apart from the signs in §**14.5** denoting units of angle and length in feet and inches, monetary unit abbreviations preceding the number, and single letter monetary unit abbreviations or symbols following the number.

14.2 Unit abbreviations are generally coded using the usual conventions of literary and mathematics braille notation, e.g. as regards the use of the letter sign and capital sign. Note that the letter sign is not required before unit abbreviations consisting of two or more lower case letters belonging to one word, e.g. cm for centimetre, but it is required where a lower case letter is followed by an upper case letter at the beginning of a unit abbreviation, e.g. mW for milliwatts.

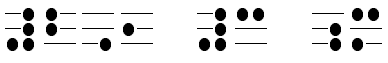
14.2.1 Capitals should normally be indicated, even in non-capitalized braille. However, conventional informal abbreviations such as MPH, M.P.H., MPG, etc., can be treated as lower case in non-capitalized braille.

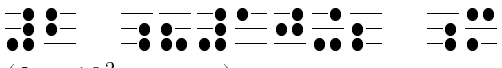
14.2.2 mmHg should be coded with a dot 6 before the abbreviation Hg for mercury, unless the special braille code for chemistry is being used (see *Braille Science Notation*).

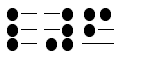
14.3 μ is coded as , Å (ångström) coded as , Ω (ohm) coded as , % (percent) coded as , £ (pounds sterling) coded as , $ (dollars) coded as , ¢ (cent) coded as , and € (euro) as . (See however, §**14.4**.)

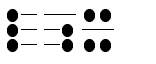
Ex.1 3 metres 

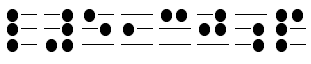
6 m
(6 metres)

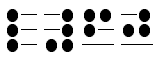
2, 3 m 
(2, 3 metres)

2×10^2 m 
(2×10^2 metres)

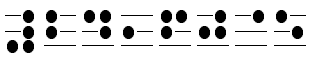
£6 
(6 pounds sterling)

£*x* 
(*x* pounds sterling)

£5.30p 
(5 pounds 30 pence)

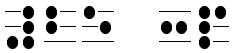
£60m 
(60 million pounds)

24.60 €



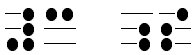
(24 euros 60 cents)

25%



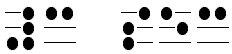
(25 percent)

3 s



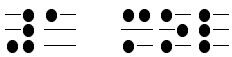
(3 seconds)

3 sec



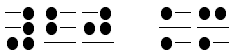
(3 seconds)

1 mol



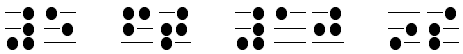
(1 mole)

20 km



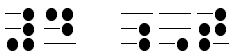
(20 kilometres)

5 ft 10 ins



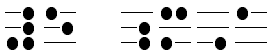
(5 feet 10 inches)

4 T



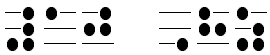
(4 teslas)

5 mA



(5 milliamperes)

10 Hz

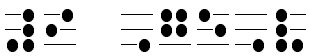


(10 hertz)

3.3 Pa

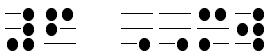
(3.3 pascals)

9 GeV



(9 gigaelectronvolts)

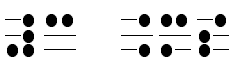
6 MW

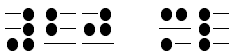


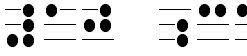
(6 megawatts)

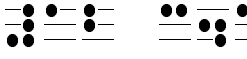
8 Å

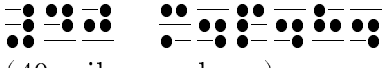
(8 ångströms)

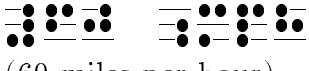
3 μs 
(3 microseconds)

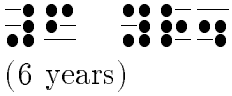
20 ml 
(20 millilitres)

10 cc 
(10 cubic centimetres)

12 c.c. 
(12 cubic centimetres)

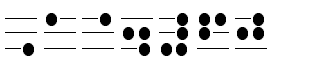
40 m.p.h. 
(40 miles per hour)

60 MPH 
(60 miles per hour)

6 yr. 
(6 years)

14.4 Unit abbreviations should generally be coded in the same way, whether or not accompanying a number.

£ (spaced) should, however, be coded as ; and $ (spaced) coded as . These signs should also be used when the symbols occur in conjunction with letters in a monetary unit. See also §**14.5.2**.

Ex.2 A$60 
(60 Australian dollars)

14.5 In simple expressions of angle, or length in feet and inches,

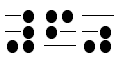
° (degrees) is coded as

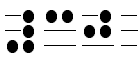
′ (minutes, or feet) as

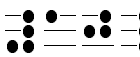
″ (seconds, or inches) as

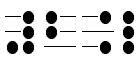
^{c} (radians, when used instead of rad) as

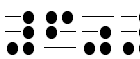
and follow the number to which they apply, with the whole group unspaced. When the degree sign follows a lower number it should be preceded by the superscript sign to avoid ambiguity.

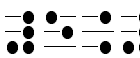
Ex.3 6° 
(6 degrees)

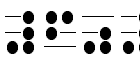
30′ 
(30 minutes, or 30 feet)

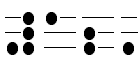
10″ 
(10 seconds, or 10 inches)

$2\pi^c$ 
(2π radians)

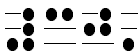
6°30′ 
(6 degrees 30 minutes)

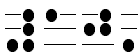
5′10″ 
(5 minutes 10 seconds, or 5 feet 10 inches)

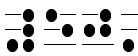
6°30′10″ 
(6 degrees 30 minutes 10 seconds)

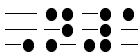
$\frac{1}{2}°$ 
($\frac{1}{2}$ degree)

14.5.1 In expressions of temperature, and in bearings, the letters C, F; N, S, E, W, are brailled unspaced from the number to which they apply.

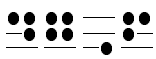
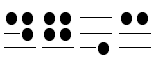
Ex.4 30°F 
(30 degrees Fahrenheit)

10°C 
(10 degrees Celsius)

50°S 
(50 degrees south)

N30°W 
(North 30° west)

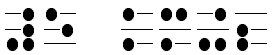
14.5.2 In combined units (see §**14.7**), °, °F and °C are coded as

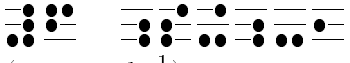
 

respectively.

These abbreviations are also used in braille when the unit symbol is not attached to a number.

14.6 Indices attached to unit abbreviations or words, should be shown as lower numbers immediately following the superscript sign.

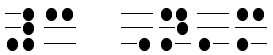
Ex.5 5 km^2 
(5 kilometre^2)

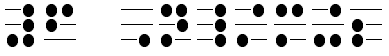
6 s^{-1} 
(6 second^{-1})

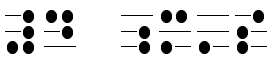
14.7 In combined units, a dot 3 should be inserted between the individual units unless an index or stroke is present at that point. A group consisting of a multiplying prefix attached to a basic unit symbol (e.g. kg, cm, etc.) is to be regarded as a single unit, and so a dot 3 separator should not intervene.

The stroke / should be brailled as .

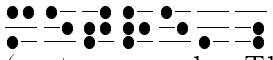
14.7.1 In print, the individual units in combined units are usually separated by spaces or half-spaces if not by a stroke (dots are also occasionally used). The dot 3 separator should, however, always be used between individual units in braille according to §**14.7**, even when no separation is shown between them in print.

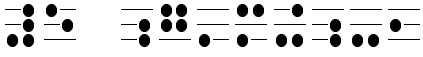
Ex.6 3 N m 
(3 newton metres)

6 N/m^2 
(6 newtons per metre^2)

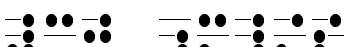
4 m s 
(4 metre seconds)

10 ms
(10 milliseconds)

metre s 
(metre seconds. This example shows the letter sign used before s for second when the dot 3 might otherwise be read as an apostrophe.)

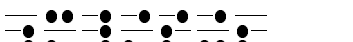
5 g m^{-1} 
(5 gram metre^{-1})

30 m/s



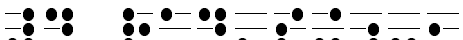
(30 metres per second)

m/s^2



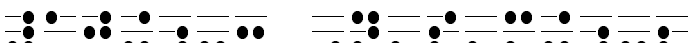
(metres per second^2)

4 rad s^{-1}



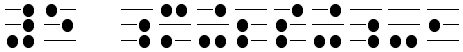
(4 radians second^{-1})

10^{-3} N s m^{-2}



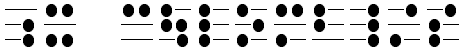
(10^{-3} newton second metre^{-2})

5 m^2 s^{-1}



(5 metre^2 second^{-1})

x coulomb/s



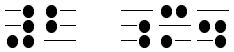
(x coulombs per second)

14.8 Long combined units may be split at the end of a braille line using the dot 5 mathematical hyphen. A dot 3, if present at that point, remains before the dot 5 hyphen. A stroke at that point should be taken onto the new line. Short unit expressions should not be divided.

It is preferable for spaced units not to be separated from their preceding number at the end of a braille line.

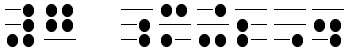
14.9 The dot 6 mathematical separation sign is not required after a unit abbreviation before following punctuation unless the abbreviation ends with an index or one of the angle symbols given in §**14.5**.

Ex.7 2 m.



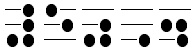
(2 metres.)

7 m^2.



(7 metre^2.)

5°.



(5 degrees.)

TABLES

(SI units and prefixes are taken from The Association for Science Education booklet: *SI Units, Signs, Symbols and Abbreviations* (1981). Other non-SI units are taken from *Nuffield Advanced Science Book of Data* (1982).)

SI UNITS

Name	**Symbol**	**Name**	**Symbol**
metre	m	coulomb	C
kilogram	kg	volt	V
second	s	ohm	Ω
ampere	A	siemens	S
kelvin	K	farad	F
candela	cd	weber	Wb
mole	mol	tesla	T
radian	rad	henry	H
steradian	sr	lumen	lm
hertz	Hz	lux	lx
newton	N	becquerel	Bq
pascal	Pa	gray	Gy
joule	J	sievert	Sv
watt	W		

MULTIPLYING PREFIXES

These may be attached to any of the above units. (Exceptionally kg already has a prefix attached, but other multiples e.g. mg, g, are formed in the obvious way.)

Sub-multiple	**Prefix**	**Symbol**	**Multiple**	**Prefix**	**Symbol**
10^{-1}	deci	d	10^{1}	deca	da
10^{-2}	centi	c	10^{2}	hecto	h
10^{-3}	milli	m	10^{3}	kilo	k
10^{-6}	micro	μ	10^{6}	mega	M
10^{-9}	nano	n	10^{9}	giga	G
10^{-12}	pico	p	10^{12}	tera	T
10^{-15}	femto	f	10^{15}	peta	P
10^{-18}	atto	a	10^{18}	exa	E

(These prefixes are also sometimes used before units in the following table.)

OTHER NON-SI UNIT SYMBOLS

Name	**Symbol**
ångström	Å
astronomical unit	au (also AU)
atmosphere	atm
atomic mass unit	u
biot	Bi
British thermal unit	Btu
calorie	cal
curie	Ci
day	d
debye	D
decibel	dB
degree (angular)	°
degree Celsius	°C
degree Fahrenheit	°F
dyne	dyn
electronvolt	eV
foot	ft (also ′)
franklin	Fr
gallon	gal
gauss	G
hectare (100 ares)	ha
horsepower	hp
hour	h
hundredweight	cwt
inch	in (also ″)
kilogram-force (kilopond)	kgf kp
knot	kn
litre	l (also L)
micron	μm (also μ)
mile (nautical)	n mile (braille as n. mile)
minute (angle)	′
minute (time)	min (m is also used in time of day, e.g. 18 h 23 m)
oersted	Oe
ounce	oz
ounce (fluid)	fl oz (A space is left between fl and oz in braille.)
pint	pt
poise	P
pound (weight)	lb
pound-force	lbf
rad or röntgen	R
rem	Rem

second (angle)	″
stokes	St
ton-force	ton f (braille as tonf)
tonne	t
torr	Torr
X unit	Xu
yard	yd
year	a

15 ALPHABETS USED IN MATHEMATICS

The following lists give the Greek and German (Gothic) alphabets. These are coded as the letters or signs indicated, preceded by the (small or capital) Greek letter fount sign, or a (small or capital) unassigned fount sign chosen from those listed in §**2.2** for German letters.

GREEK

Small	**Capital**			**Small**	**Capital**		
α	A	a	alpha	ν	N	n	nu
β	B	b	beta	ξ	Ξ	x	xi
γ	Γ	g	gamma	o	O	o	omicron
δ	Δ	d	delta	π	Π	p	pi
ϵ	E	e	epsilon	ρ, ϱ	P	r	rho
ζ	Z	z	zeta	σ, ς	Σ	s	sigma
η	H	⠱	eta	τ	T	t	tau
θ, ϑ	Θ	⠹	theta	υ	Υ	u	upsilon
ι	I	i	iota	ϕ, φ	Φ	f	phi
κ	K	k	kappa	χ	X	⠯	chi
λ	Λ	l	lambda	ψ	Ψ	y	psi
μ	M	m	mu	ω	Ω	w	omega

GERMAN

Small	Capital		Small	Capital	
$\mathfrak{a}$	$\mathfrak{A}$	a	$\mathfrak{n}$	$\mathfrak{N}$	n
$\mathfrak{b}$	$\mathfrak{B}$	b	$\mathfrak{o}$	$\mathfrak{O}$	o
$\mathfrak{c}$	$\mathfrak{C}$	c	$\mathfrak{p}$	$\mathfrak{P}$	p
$\mathfrak{d}$	$\mathfrak{D}$	d	$\mathfrak{q}$	$\mathfrak{Q}$	q
$\mathfrak{e}$	$\mathfrak{E}$	e	$\mathfrak{r}$	$\mathfrak{R}$	r
$\mathfrak{f}$	$\mathfrak{F}$	f	$\mathfrak{s}$	$\mathfrak{S}$	s
$\mathfrak{g}$	$\mathfrak{G}$	g	$\mathfrak{t}$	$\mathfrak{T}$	t
$\mathfrak{h}$	$\mathfrak{H}$	h	$\mathfrak{u}$	$\mathfrak{U}$	u
$\mathfrak{i}$	$\mathfrak{I}$	i	$\mathfrak{v}$	$\mathfrak{V}$	v
$\mathfrak{j}$	$\mathfrak{J}$	j	$\mathfrak{w}$	$\mathfrak{W}$	w
$\mathfrak{k}$	$\mathfrak{K}$	k	$\mathfrak{x}$	$\mathfrak{X}$	x
$\mathfrak{l}$	$\mathfrak{L}$	l	$\mathfrak{y}$	$\mathfrak{Y}$	y
$\mathfrak{m}$	$\mathfrak{M}$	m	$\mathfrak{z}$	$\mathfrak{Z}$	z

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